ME 3304 Heat Transfer

Lecture Note (7) Radiation - Chapter 12 & 13

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Heat Transfer Modes – Chapter 1

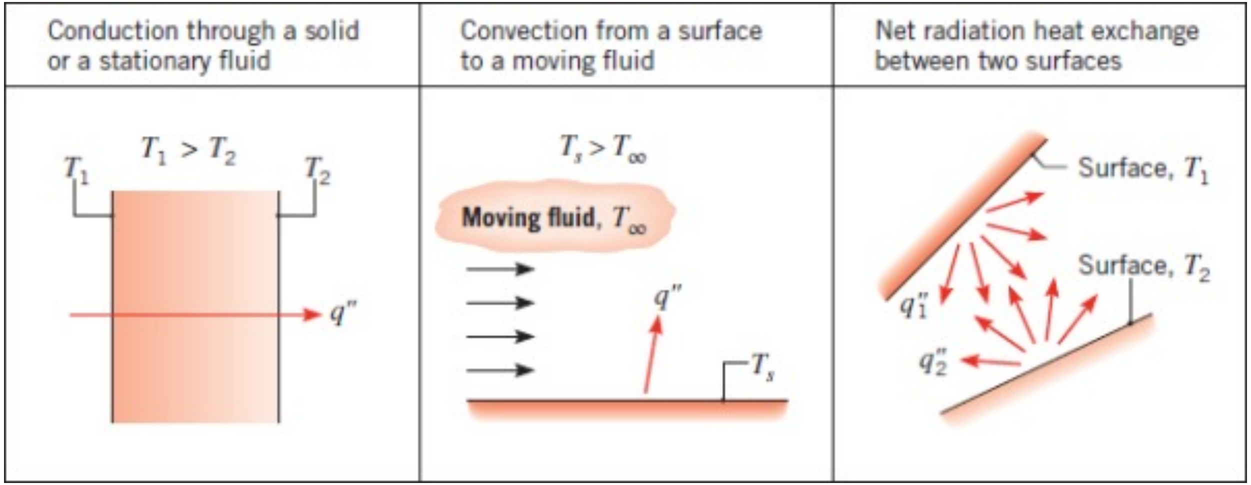


FIGURE 1.1 Conduction, convection, and radiation heat transfer modes.

Introduction to Radiation – Chapter 1

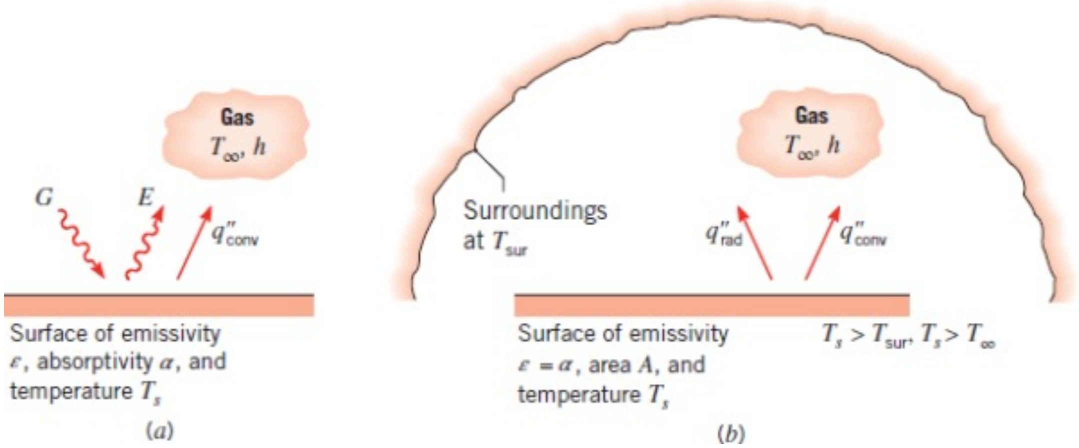


FIGURE 1.6 Radiation exchange: (a) at a surface and (b) between a surface and large surroundings.

Radiation $E_b = \sigma T_s^4$

where T_s is the *absolute temperature* (K) of the surface and σ is the *Stefan–Boltzmann constant* ($\sigma = 5.67 \times 10^{-8}$ W/m$^2 \cdot$ K^4). Such a surface is called an ideal radiator or *blackbody*.

$$E_{real\ surface} = \varepsilon \sigma T_s^4$$

where ε is a radiative property of the surface termed the emissivity. With values in the range $0 \leq \varepsilon \leq 1$, this property provides a measure of how efficiently a surface emits energy relative to a blackbody. It depends strongly on the surface material and finish.

Introduction to Radiation – Chapter 1

The rate at which radiant energy is *absorbed* per unit surface area may be evaluated from knowledge of a surface radiative property termed the *absorptivity* α, *where* $0 \leq \alpha \leq 1$.

$$G_{abs} = \alpha G \qquad \text{where, } G = \sigma T_{sur}^4$$

If the surface is assumed to be one for which $\alpha = \varepsilon$ (a *gray surface*), the *net* rate of radiation heat transfer *from* the surface, expressed per unit area of the surface, is

$$q''_{rad} = \frac{q}{A} = \varepsilon E_b - \alpha G = \varepsilon\sigma(T_s^4 - T_{sur}^4)$$

For many applications, it is convenient to express the net radiation heat exchange in the form

$$q_{rad} = h_r A(T_s - T_{sur})$$

where, h_r : *radiation heat transfer coefficient*

$$h_r \equiv \varepsilon\sigma(T_s + T_{sur})(T_s^2 - T_{sur}^2)$$

Chapter 12 Radiation – Process & Properties

Thermal Radiation

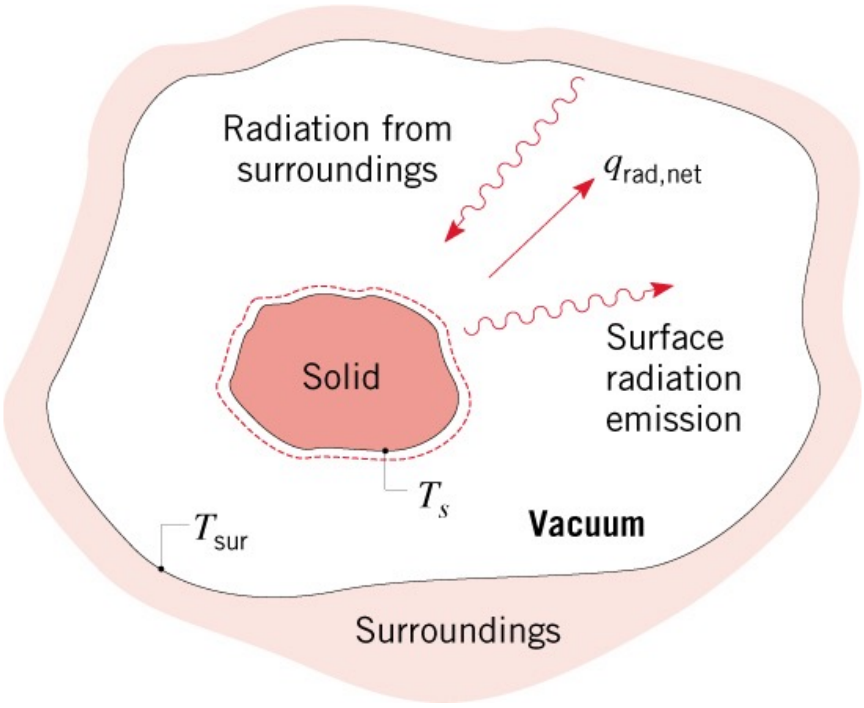


FIGURE 12.1 Radiation cooling of a hot solid

Heat transfer by conduction and convection requires the presence of a temperature gradient in some form of matter. In contrast, heat transfer by *thermal radiation* requires no matter.

The mechanism of emission is related to energy released as a result of oscillations or transitions of the many electrons that constitute matter.

One theory views radiation as the propagation of a collection of particles termed photons or quanta. Alternatively, radiation may be viewed as the propagation of electromagnetic waves.

$$\lambda = \frac{c}{v}$$

where, *wavelength* λ, *frequency* v and c is the *speed of light* in the medium.
(For propagation in a vacuum, $c_o = 2.998 \times 10^8$ m/s. The unit of wavelength is commonly the micrometer (μm), where 1 μm = 10^{-6} m.)

Thermal Radiation

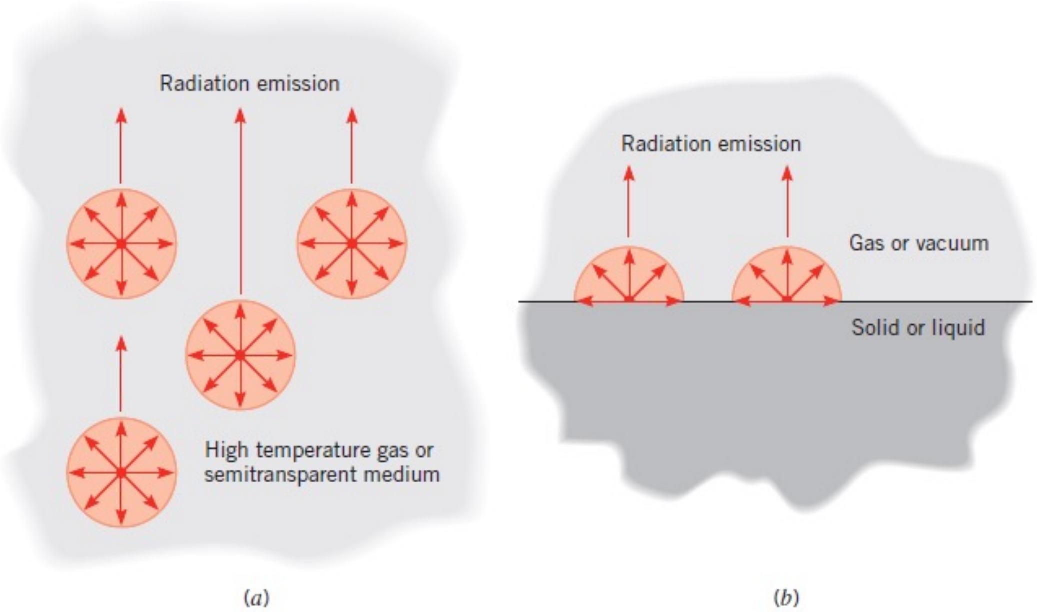


FIGURE 12.2 The emission process. (*a*) As a volumetric phenomenon. (*b*) As a surface phenomenon.

In most solids and liquids, radiation emitted from interior molecules is strongly absorbed by adjoining molecules. It is for this reason that emission from a solid or a liquid into an adjoining gas or a vacuum can be viewed as a surface phenomenon, except in situations involving nanoscale or microscale devices.

Thermal Radiation

All of the visible and infrared (IR) (0.1 to 100 μm), that is termed *thermal radiation* because it is both caused by and affects the thermal state or temperature of matter.

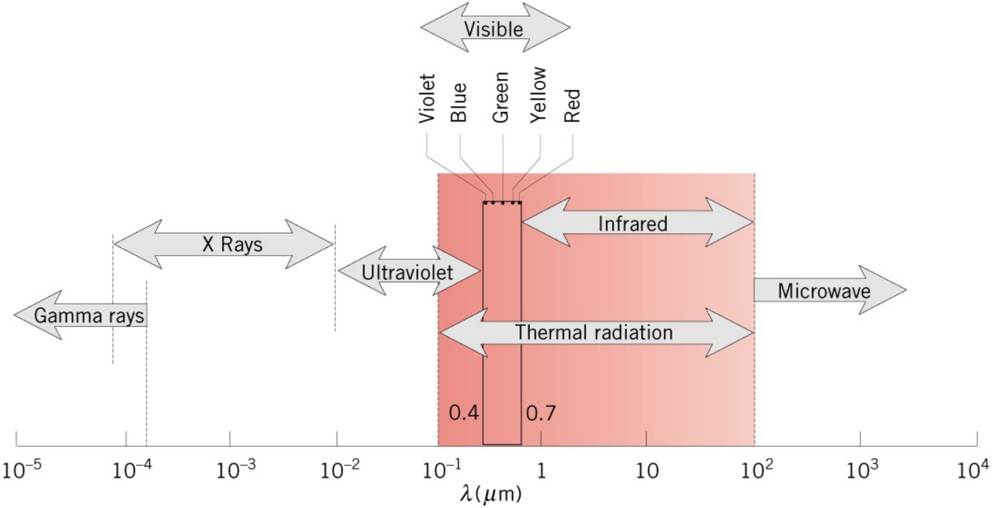


FIGURE 12.3 Spectrum of electromagnetic radiation.

The magnitude of the radiation varies with wavelength, and the term *spectral* is used to refer to the nature of this dependence. The second feature relates to its *directionality*.

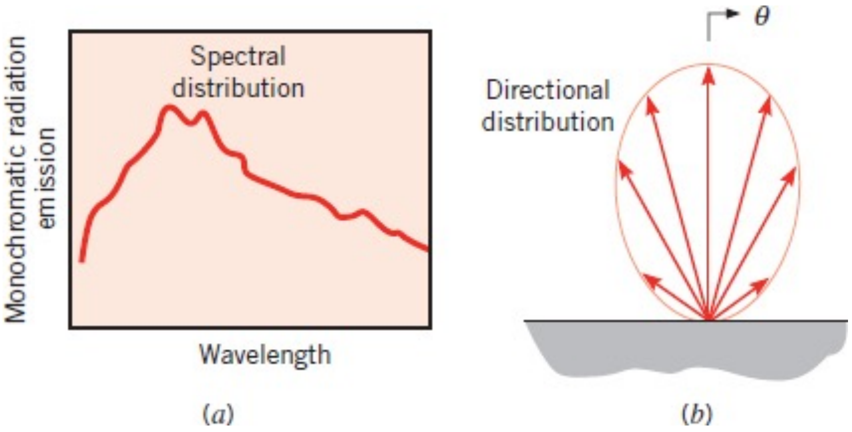
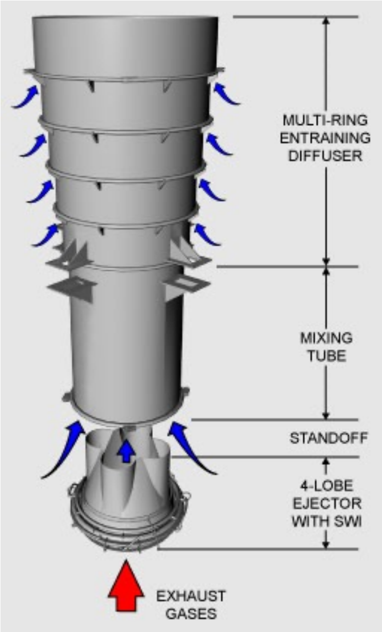
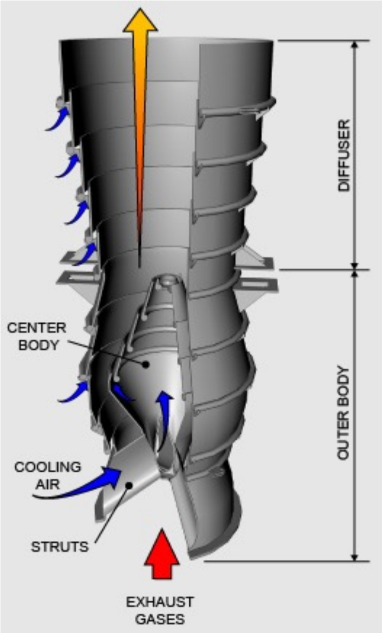


FIGURE 12.4 Radiation emitted by a surface. (a) Spectral distribution. (b) Directional distribution.

Thermal Radiation

CH 47 “Chinook”





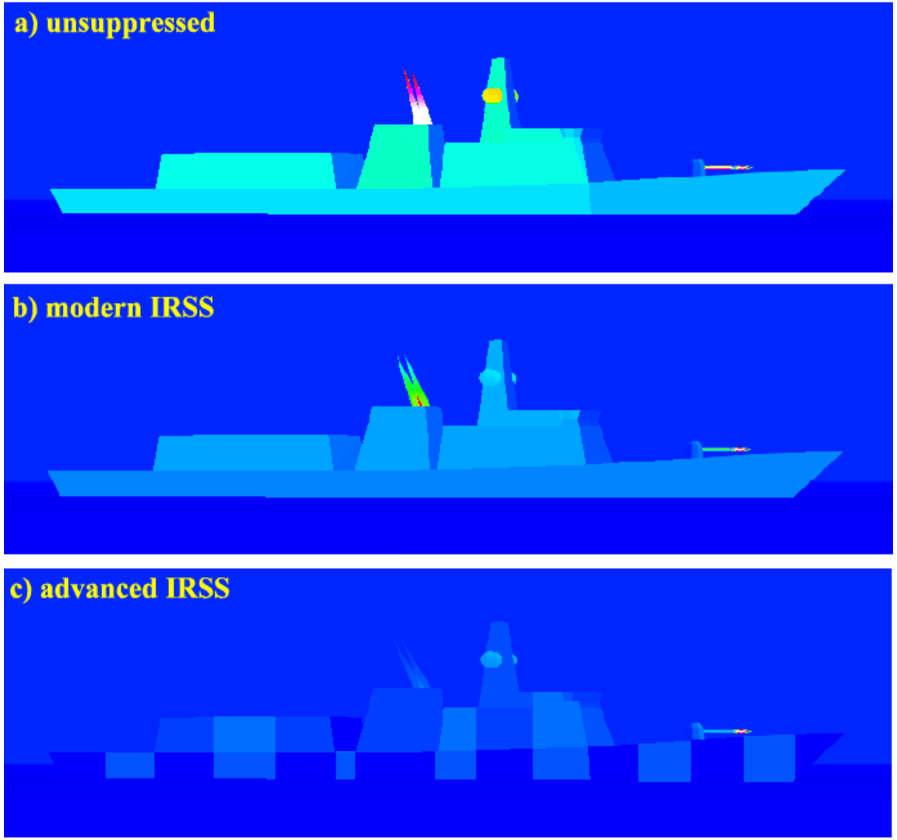
WR Davis Engineering
Engine Exhaust IR Suppression

Thermal Radiation

Navy Stealth Destroyer

USS Arleigh Burke-class Destroyer

Thermal Radiation



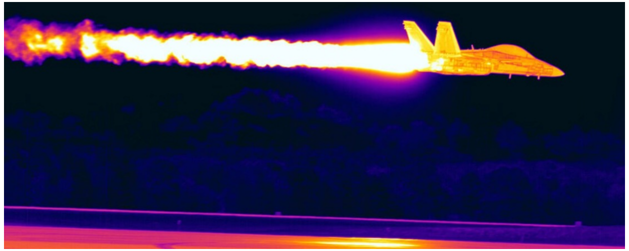
WR Davis Engineering

Thermal Radiation

F15 Eagle

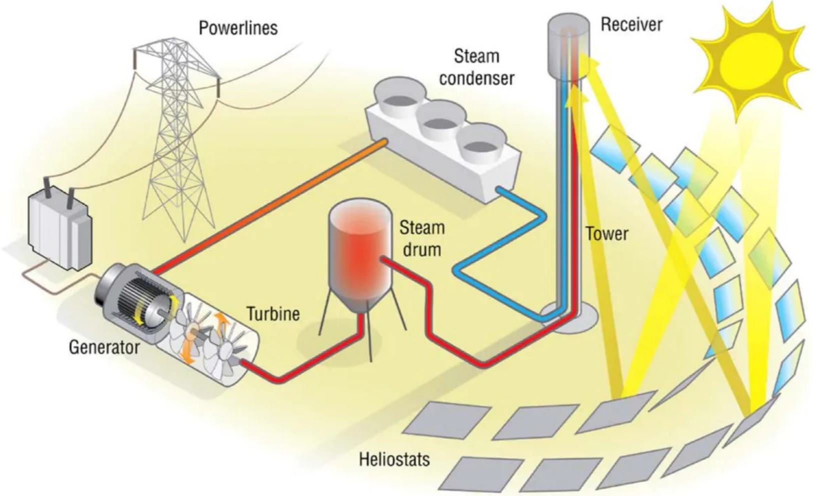
F4 Phantom

F-117 Nighthawk



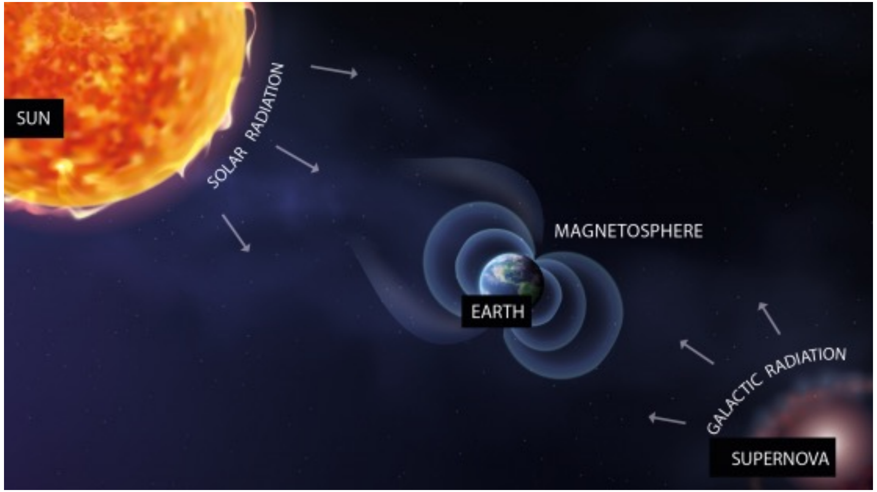
FLIR.com

Thermal Radiation



Crescent Dunes (Nevada) Solar Energy
110 MW concentrating solar power electricity plant

Radiation – Cosmic Rays



Cosmic rays are extremely high-energy subatomic particles – mostly protons and atomic nuclei accompanied by electromagnetic emissions – that move through space, eventually bombarding the Earth's surface. They travel at nearly the speed of light, which is approximately 300,000 kilometers per second.

On average, **people** are exposed to around 3.5 millisieverts of radiation per year.

About half of this comes from artificial sources such as X-ray, mammography and CT scans, while the other half we get from natural sources, of which about 10 per cent comes from cosmic radiation. Sievert is the measure of health risk from radiation: one sievert carries with it a 5.5 per cent chance of eventually developing radiation-induced cancer later in life.

Aircrew will hardly exceed a dose of one millisievert per year. Crew serving on long-haul polar routes, though, might be exposed to an annual effective dose of up to six millisieverts.

Astronauts would get the same dose in 12 days, as aircrew gets in a year.

Credit to IAEA

Radiation Heat Fluxes

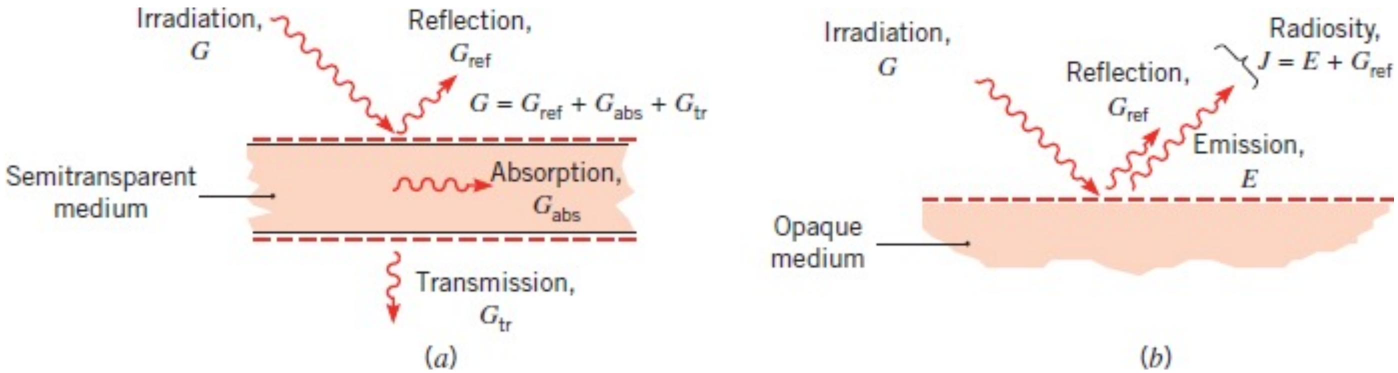


FIGURE 12.5 Radiation at a surface. (*a*) Reflection, absorption, and transmission of irradiation for a semitransparent medium. (*b*) The radiosity for an opaque (not transparent) medium.

ρ → reflectivity → fraction of irradiation (G) reflected.
α → absorptivity → fraction of irradiation absorbed.
τ → transmissivity → fraction of irradiation transmitted through the medium.

$\rho + \alpha + \tau = 1$ for any medium. $\rho + \alpha = 1$ for an opaque medium.

Radiation Heat Fluxes

Table 12.1

Flux (W/m^2)	Description	Comment
Emissive power, E	Rate at which radiation is emitted from a surface per unit area	$E = \varepsilon\sigma T_s^4$
Irradiation, G	Rate at which radiation is incident upon a surface per unit area	Irradiation can be reflected, absorbed, or transmitted
Radiosity, J	Rate at which radiation leaves a surface per unit area	For an opaque surface $J = E + \rho G$
Net radiative flux, $q''_{rad} = J - G$	Net rate of radiation leaving a surface per unit area	For an opaque surface $q''_{rad} = \varepsilon\sigma T_s^4 - \alpha G$

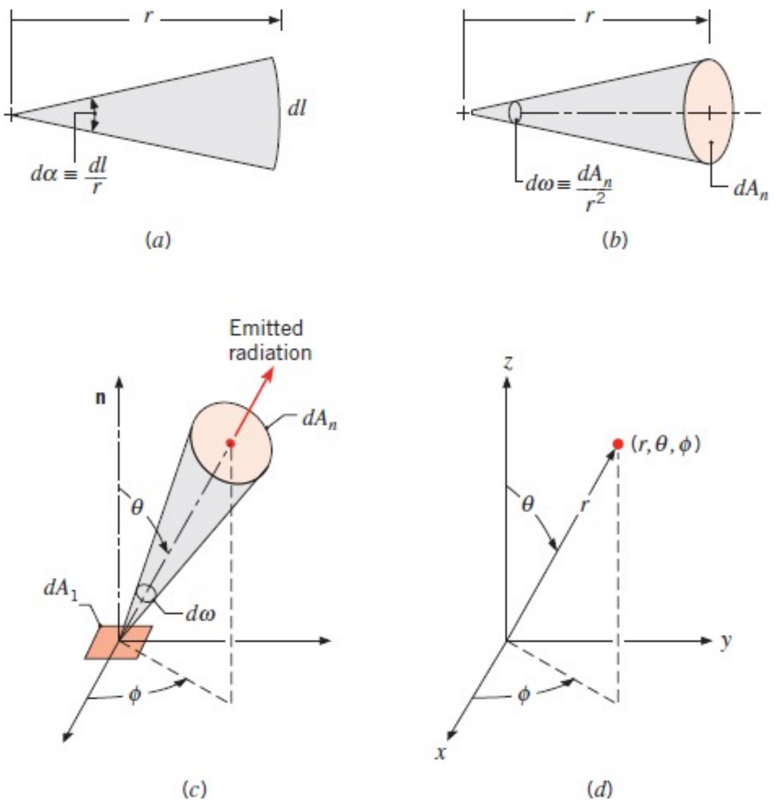
Radiation Intensity

Radiation that leaves a surface can propagate in all possible directions and we are often interested in knowing its directional distribution.

Also, radiation incident upon a surface may come from different directions, and the manner in which the surface responds to this radiation depends on the direction.

Such directional effects can be of primary importance in determining the net radiative heat transfer rate and may be treated by introducing the concept of *radiation intensity*.

Radiation Intensity



The amount of radiation emitted from a surface, dA_1 and propagating in a particular direction, θ, ϕ is quantified in terms of a differential solid angle associated with the direction.

$$d\omega \equiv \frac{dA_n}{r^2}$$

dA_n : unit element of surface on a hypothetical sphere and normal to the θ, ϕ direction.

FIGURE 12.6 Mathematical definitions. (*a*) Plane angle. (*b*) Solid angle. (*c*) Emission of radiation from a differential area dA_1 into a solid angle $d\omega$ subtended by dA_n at a point on dA_1. (*d*) The spherical coordinate system.

Radiation Intensity

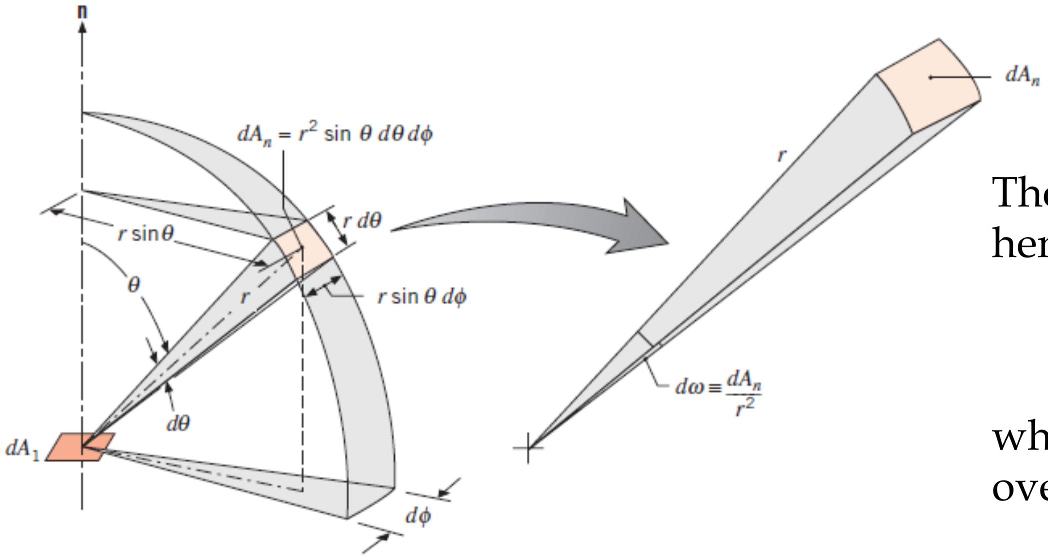


FIGURE 12.7 The solid angle subtended by dA_n at a point on dA_1 in the spherical coordinate system.

$$dA_n = r^2 \sin\theta d\theta d\phi$$

$$d\omega = \frac{dA_n}{r^2} = \sin\theta d\theta d\phi$$

The solid angle associated with a complete hemisphere is

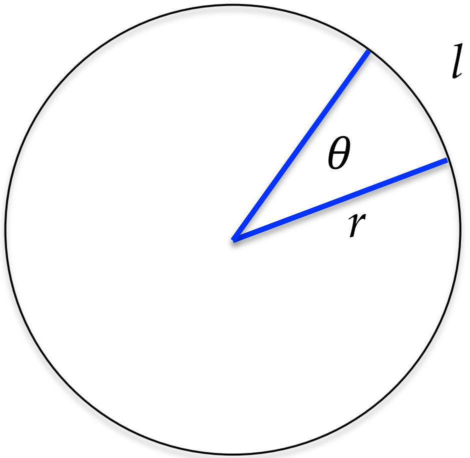
$$\omega_{\text{hemi}} = \int_0^{2\pi} \int_0^{\pi/2} \sin\theta d\theta d\phi = 2\pi \text{ sr}$$

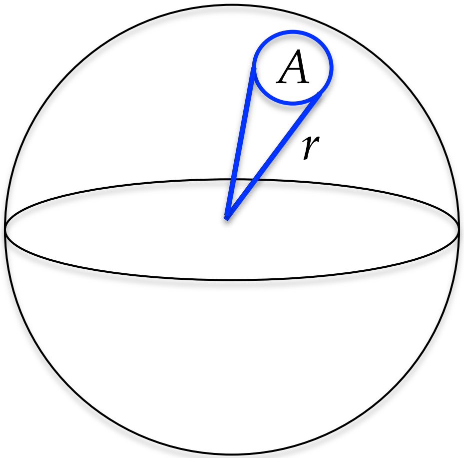
where the symbol *hemi* refers to integration over the hemisphere.

Note that the unit of the solid angle (planar angle), ω is the steradian (sr), analogous to radians for plane angles.

Radiation Intensity

Steradian (sr) is the solid angle that subtends an $A = R^2$ on the surface of sphere.





$$\theta = \frac{l}{r}$$

Circle = 2π [radians]

$$\omega = \frac{A}{r^2}$$

Sphere = 4π [steradians]

Radiation Intensity

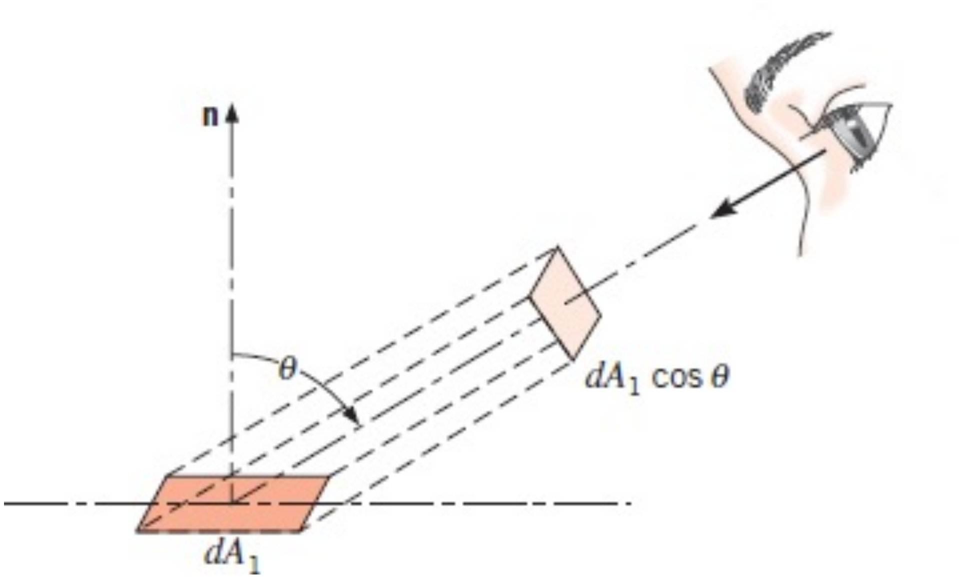


FIGURE 12.8 The projection of dA_1 normal to the direction of radiation.

Define $I_{\lambda,e}$ (spectral intensity) as the rate at which radiant energy is emitted at the wavelength λ in the (θ, ϕ) direction, per unit area of the emitting surface normal to this direction, per unit solid angle about this direction, and per unit wavelength interval dλ about λ.

$$I_{\lambda,e}(\lambda,\theta,\phi) \equiv \frac{dq}{(dA_1 \cos\theta)\cdot d\omega \cdot d\lambda} \quad [W/m^2 \cdot sr \cdot \mu m]$$

where $(dq/d\lambda) \equiv dq_\lambda$ is the rate at which radiation of wavelength λ leaves dA_1 and passes through dA_n. Rearranging the above, it follows that

$$dq_\lambda \equiv \frac{dq}{d\lambda} = I_{\lambda,e}(\lambda,\theta,\phi)\, dA_1 \cos\theta\, d\omega$$

Radiation Intensity

$$dq_\lambda \equiv \frac{dq}{d\lambda} = I_{\lambda,e}(\lambda,\theta,\phi)\, dA_1 \cos\theta\, d\omega$$

where dq_λ has the units of W/μm

Expressing the above equation per unit area of the emitting surface and substituting from $d\omega = sin\theta d\theta d\phi$,

the spectral radiation *flux* associated with dA_1 is

$$dq''_\lambda = I_{\lambda,e}(\lambda,\theta,\phi)\cos\theta\, d\omega = I_{\lambda,e}(\lambda,\theta,\phi)\cos\theta\sin\theta\, d\theta\, d\phi$$

Radiation Intensity

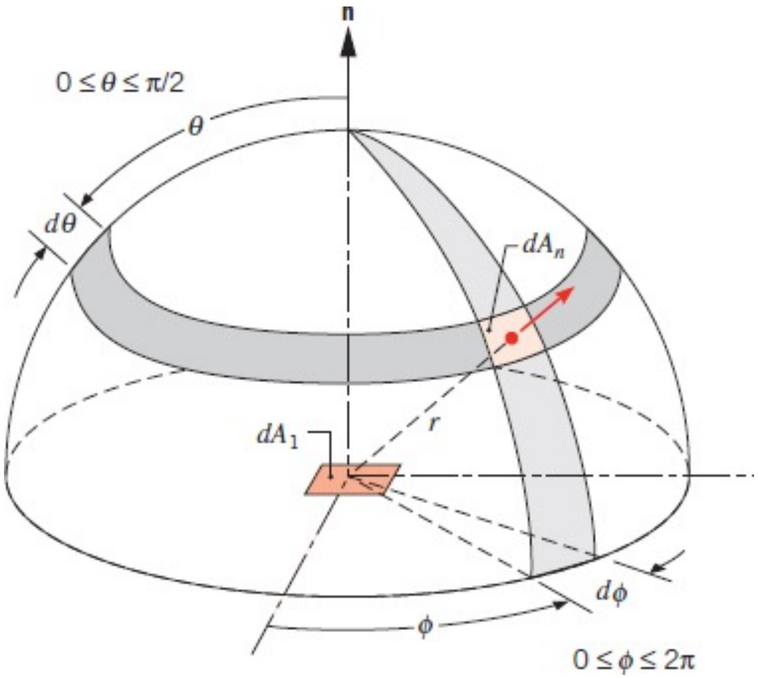


FIGURE 12.9 Emission from a differential element of area dA_1 into a hypothetical hemisphere centered at a point on dA_1.

The spectral (hemispherical) emissive power ($W/m^2 \cdot \mu m$) corresponds to spectral emission over all possible directions.

$$E_\lambda(\lambda) = \int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,e}(\lambda,\theta,\phi)\cos\theta\sin\theta d\theta d\phi$$

The total (hemispherical) emissive power (W/m^2) corresponds to emission over all directions and wavelengths.

$$E = \int_0^{\infty}\int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,e}(\lambda,\theta,\phi) cos\theta sin\theta d\theta d\phi d\lambda$$

For a diffuse surface(emitter), emission is isotropic so that

$$E_\lambda(\lambda) = \pi I_{\lambda,e}(\lambda) \quad \text{or} \quad E = \pi I_e$$

where I_e is the *total intensity* of the emitted radiation. Note that the π has the *unit steradians*.

Irradiation

The concepts may be extended to *incident* radiation. Such radiation may originate from emission and reflection occurring at other surfaces and will have spectral and directional distributions determined by the spectral intensity $I_{\lambda,i}\ (\lambda, \theta, \phi)$

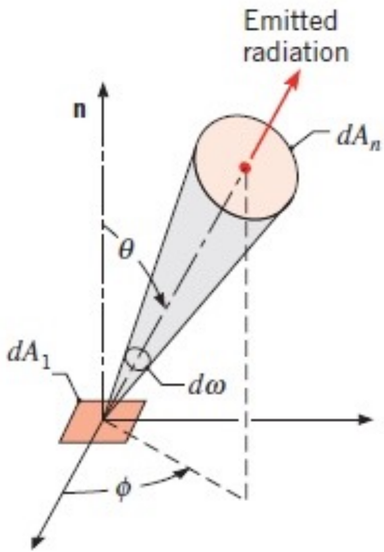


FIGURE 12.6 Emission of radiation from a differential area dA_1 into a solid angle $d\omega$ subtended by dA_n at a point on dA_1.

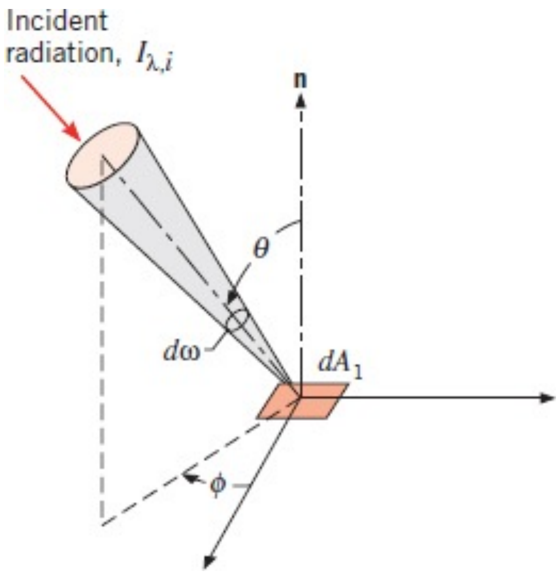


FIGURE 12.10 Directional nature of incident radiation.

Irradiation

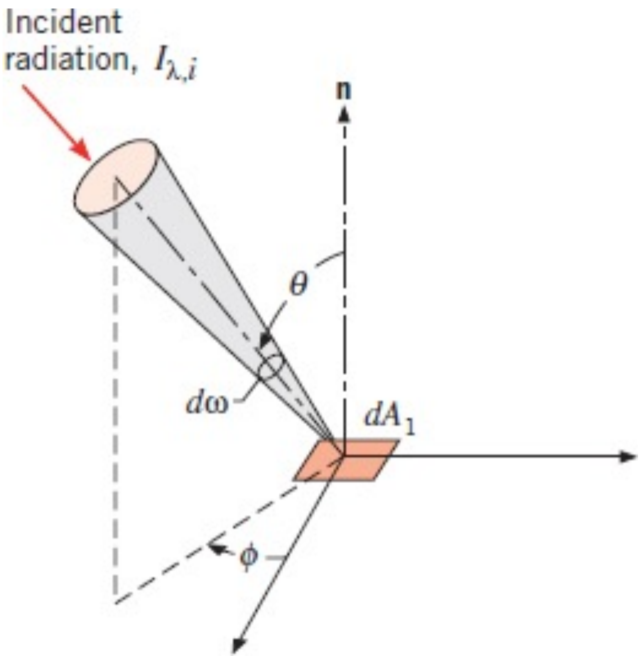


FIGURE 12.10 Directional nature of incident radiation.

The spectral (hemispherical) irradiation $(W/m^2 \cdot \mu m)$ corresponds to radiation incident from all possible directions.

$$G_\lambda = \int_0^{2\pi} \int_0^{\pi/2} I_{\lambda,i}(\lambda, \theta, \phi) cos\theta sin\theta d\theta d\phi$$

The total (hemispherical) irradiation (W/m^2) corresponds to radiation incident over all directions and wavelengths.

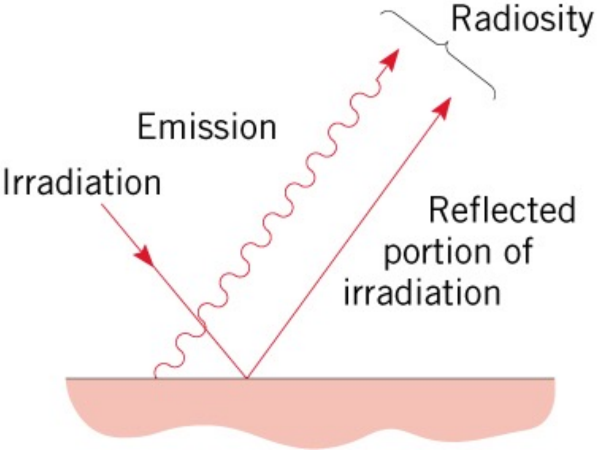
$$G = \int_0^{\infty} \int_0^{2\pi} \int_0^{\pi/2} I_{\lambda,i}(\lambda, \theta, \phi) cos\theta sin\theta d\theta d\phi d\lambda$$

For a diffuse surface, incident radiation is isotropic so that

$G_\lambda(\lambda) = \pi I_{\lambda,i}(\lambda)$ so that $G = \pi I_i$

Radiosity

The radiosity of an opaque surface accounts for all of the radiation leaving the surface in all directions and may include contributions from both reflection and emission.



With $I_{\lambda,e+r}$ designating the spectral intensity associated with radiation emitted by the surface and the reflection of incident radiation, the spectral radiosity is:

$$J_\lambda(\lambda) = \int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,e+r}(\lambda,\theta,\phi)\cos\theta\sin\theta d\theta d\phi$$

and the total radiosity is

$$J = \int_0^{\infty}\int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,e+r}(\lambda,\theta,\phi) cos\theta sin\theta d\theta d\phi d\lambda$$

Blackbody Radiation

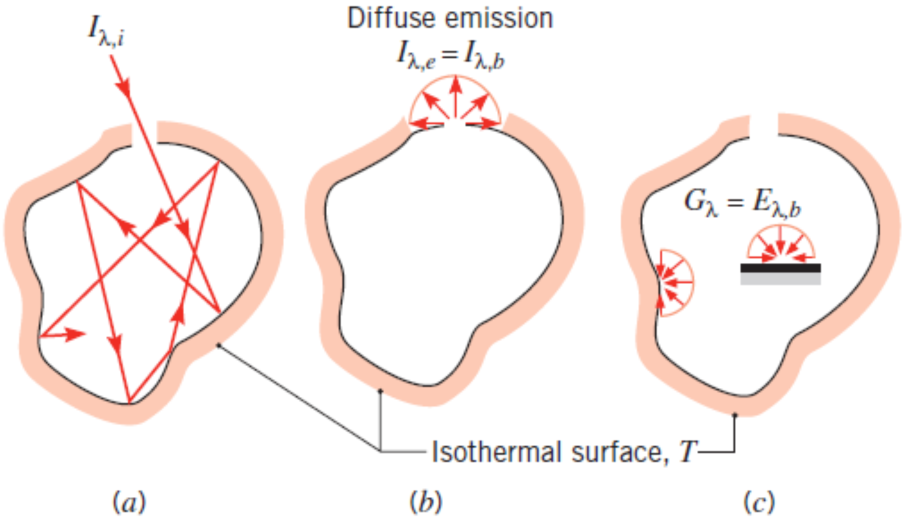


FIGURE 12.11 Characteristics of an isothermal blackbody cavity. (a) Complete absorption. (b) Diffuse emission from an aperture. (c) Diffuse irradiation of interior surfaces.

The concept of a blackbody

1. A blackbody absorbs all incident radiation, regardless of wavelength and direction.

2. For a prescribed temperature and wavelength, no surface can emit more energy than a blackbody.

3. Although the radiation emitted by a blackbody is a function of wavelength and temperature, it is independent of direction. That is, the blackbody is a diffuse emitter.

As the perfect absorber and emitter, the blackbody serves as a standard against which the radiative properties of actual surfaces may be compared.

Blackbody Radiation – Plank Distribution

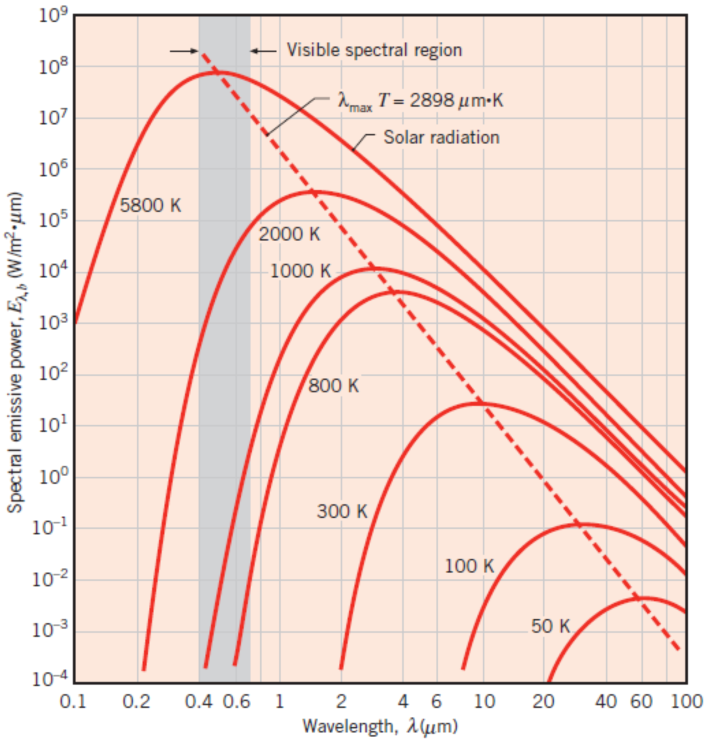


FIGURE 12.12 Spectral blackbody emissive power

The spectral distribution of the <u>blackbody emissive power</u> (determined theoretically and confirmed experimentally) is

$$E_{\lambda,b}(\lambda,T) = \pi I_{\lambda,b}(\lambda,T) = \frac{C_1}{\lambda^5\left[\exp\left(C_2/\lambda T\right)-1\right]}$$

First radiation constant: $C_1 = 3.742\times10^8\,\mathrm{W}\cdot\mu\mathrm{m}^4/\mathrm{m}^2$

Second radiation constant: $C_2 = 1.439\times10^4\,\mu\mathrm{m}\cdot\mathrm{K}$

The distribution is characterized by a maximum for which λ_{max} is given by Wien's displacement law:

$$\lambda_{max}T = C_3 = 2898\ \mu\mathrm{m}\cdot\mathrm{K}$$

Blackbody Radiation – Stefan-Boltzmann Law

The total emissive power of a blackbody is obtained by integrating the Planck distribution over all wavelengths.

$$E_b = \pi I_b = \int_0^\infty E_{\lambda,b} d\lambda = \sigma T^4$$

The results is the Stefan-Boltzmann law, where

$$\sigma = 5.670 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4} \rightarrow \text{ the Stefan-Boltzmann constant}$$

Blackbody Radiation – Band Emission

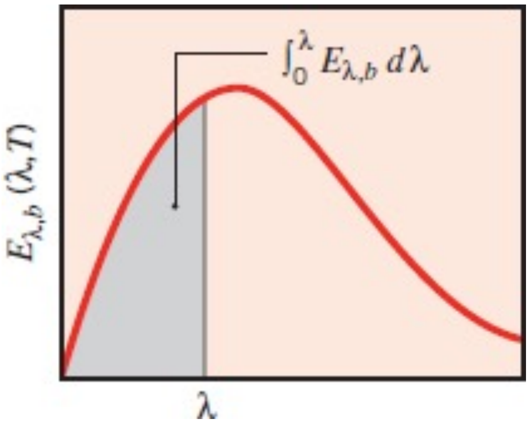


FIGURE 12.13 Radiation emission from a blackbody in the spectral band 0 to λ.

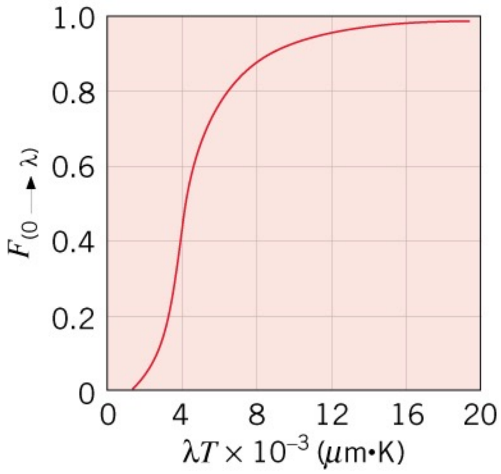


FIGURE 12.14 Fraction of the total blackbody emission in the spectral band from 0 to λ as a function of λT.

The fraction of total blackbody emission that is in a prescribed wavelength interval or band is

$$F_{(0\to\lambda)} \equiv \frac{\int_0^\lambda E_{\lambda,b}\, d\lambda}{\int_0^\infty E_{\lambda,b}\, d\lambda} = \frac{\int_0^\lambda E_{\lambda,b}\, d\lambda}{\sigma T^4}$$

$$= \int_0^\lambda \frac{E_{\lambda,b}}{\sigma T^5}\, d(\lambda T) = f(\lambda T)$$

TABLE 12.2 Blackbody Radiation Functions

λT (μm · K)	$F_{(0\rightarrow\lambda)}$	$I_{\lambda,b}(\lambda,T)/\sigma T^5$ (μm · K · sr)$^{-1}$	$\dfrac{I_{\lambda,b}(\lambda,T)}{I_{\lambda,b}(\lambda_{max},T)}$
200	0.000000	0.375034×10^{-27}	0.000000
400	0.000000	0.490335×10^{-13}	0.000000
600	0.000000	0.104046×10^{-8}	0.000014
800	0.000016	0.991126×10^{-7}	0.001372
1,000	0.000321	0.118505×10^{-5}	0.016406
1,200	0.002134	0.523927×10^{-5}	0.072534
1,400	0.007790	0.134411×10^{-4}	0.186082
1,600	0.019718	0.249130	0.344904
1,800	0.039341	0.375568	0.519949
2,000	0.066728	0.493432	0.683123
2,200	0.100888	0.589649×10^{-4}	0.816329
2,400	0.140256	0.658866	0.912155
2,600	0.183120	0.701292	0.970891
2,800	0.227897	0.720239	0.997123
2,898	0.250108	0.722318×10^{-4}	1.000000
3,000	0.273232	0.720254×10^{-4}	0.997143
3,200	0.318102	0.705974	0.977373
3,400	0.361735	0.681544	0.943551
3,600	0.403607	0.650396	0.900429
3,800	0.443382	0.615225×10^{-4}	0.851737
4,000	0.480877	0.578064	0.800291
4,200	0.516014	0.540394	0.748139
⋮	⋮	⋮	⋮

Emission from Real Surface (vs Blackbody)

“The blackbody is an ideal emitter in the sense that no surface can emit more radiation than a blackbody at the same temperature.”

A surface radiative property known as the *emissivity* may then be defined as <u>the *ratio* of the radiation emitted by the surface to the radiation emitted by a blackbody at the same temperature</u>

$$\varepsilon(T) \equiv \frac{E(T)}{E_b(T)}$$

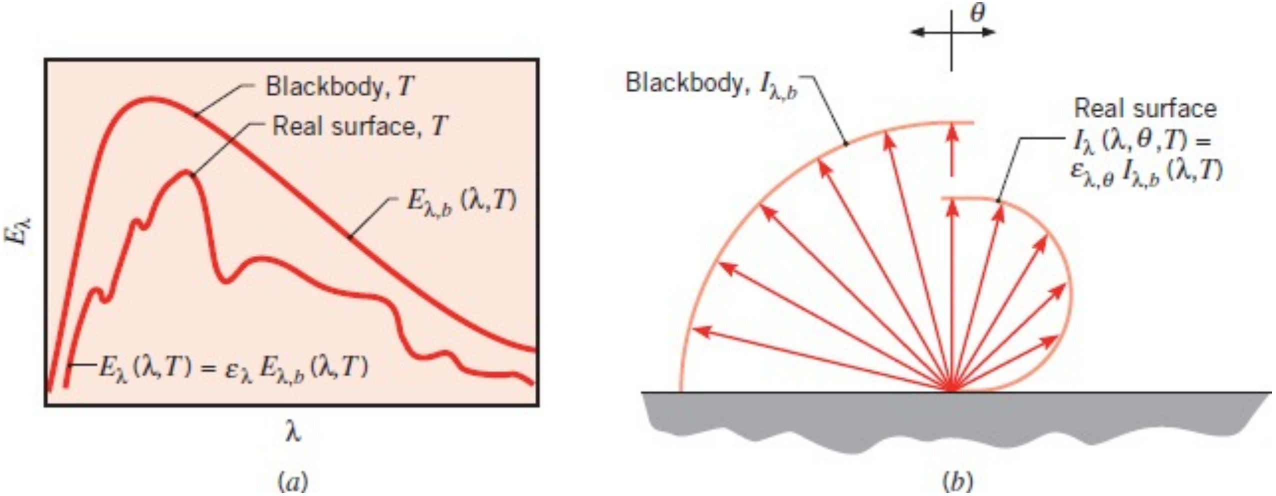


FIGURE 12.15 Comparison of blackbody and real surface emission. (a) Spectral distribution. (b) Directional distribution

Emission from Real Surface (vs Blackbody)

Total, hemispherical emissivity
(directional and spectral average)

$$\varepsilon(T) \equiv \frac{E(T)}{E_b(T)}$$

Spectral, directional emissivity

$$\varepsilon_{\lambda,\theta}(\lambda,\theta,\phi,T) \equiv \frac{I_{\lambda,e}(\lambda,\theta,\phi,T)}{I_{\lambda,b}(\lambda,\theta,\phi,T)}$$

Total, directional emissivity
(spectral average)

$$\varepsilon_{\theta}(\theta,\phi,T) \equiv \frac{I_e(\theta,\phi,T)}{I_b(\theta,\phi,T)}$$

Spectral, hemispherical emissivity
(directional average)

$$\varepsilon_{\lambda}(\lambda,T) \equiv \frac{E_\lambda(\lambda,T)}{E_{\lambda,b}(\lambda,T)}$$

Emission from Real Surface (vs Blackbody)

Spectral, hemispherical emissivity $$\varepsilon_\lambda(\lambda, T) \equiv \frac{E_\lambda(\lambda, T)}{E_{\lambda,b}(\lambda, T)}$$

$$\varepsilon_\lambda(\lambda,T) \equiv \frac{E_\lambda(\lambda,T)}{E_{\lambda,b}(\lambda,T)} = \frac{\int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,e}(\lambda,\theta,\phi,T)\cos\theta\sin\theta d\theta d\phi}{\int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,b}(\lambda,T)\cos\theta\sin\theta d\theta d\phi}$$

Total , hemispherical emissivity $$\varepsilon(T) \equiv \frac{E(T)}{E_b(T)} = \frac{\int_0^\infty \varepsilon_\lambda(\lambda,T) E_{\lambda,b}(\lambda,T) d,\lambda}{E_b(T)}$$

Emission from Real Surface

The ratio rarely falls outside the range $1.0 \le (\varepsilon/\varepsilon_n) \le 1.3$ for conductors and the range of $0.95 \le (\varepsilon/\varepsilon_n) \le 1.0$ for nonconductors. Hence to a reasonable approximation, $\varepsilon \sim \varepsilon_n$

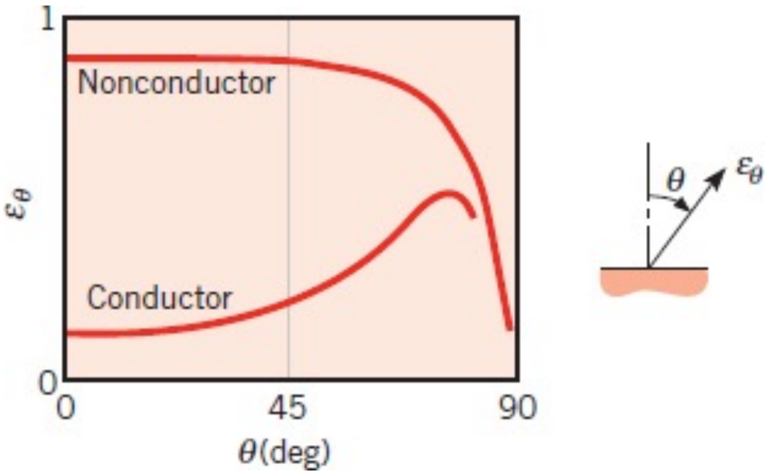


FIGURE 12.16 Representative directional distributions of the total, directional emissivity

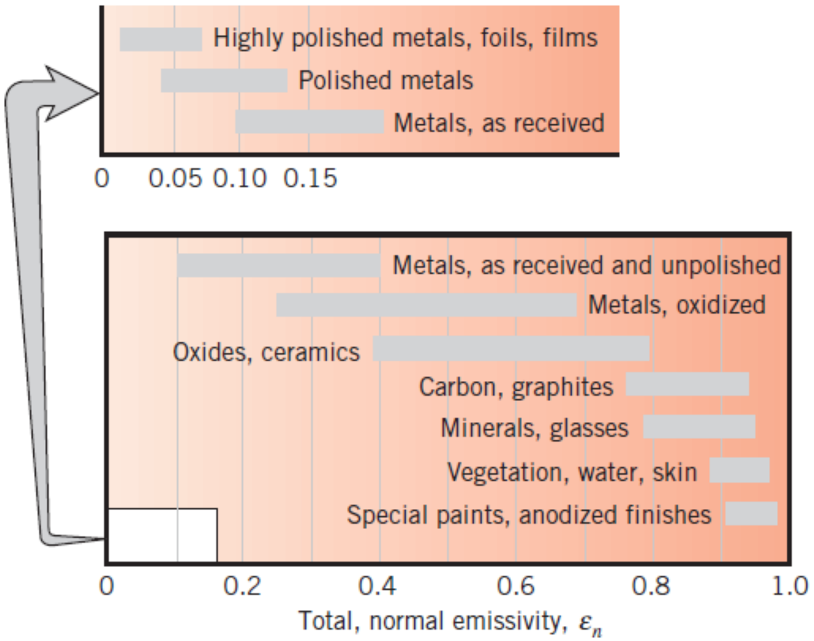


FIGURE 12.19 Representative values of the total, normal emissivity ε_n

Emission from Real Surface

Note decreasing $\varepsilon_{\lambda,n}$ with increasing λ for metals and different behavior for nonmetals.

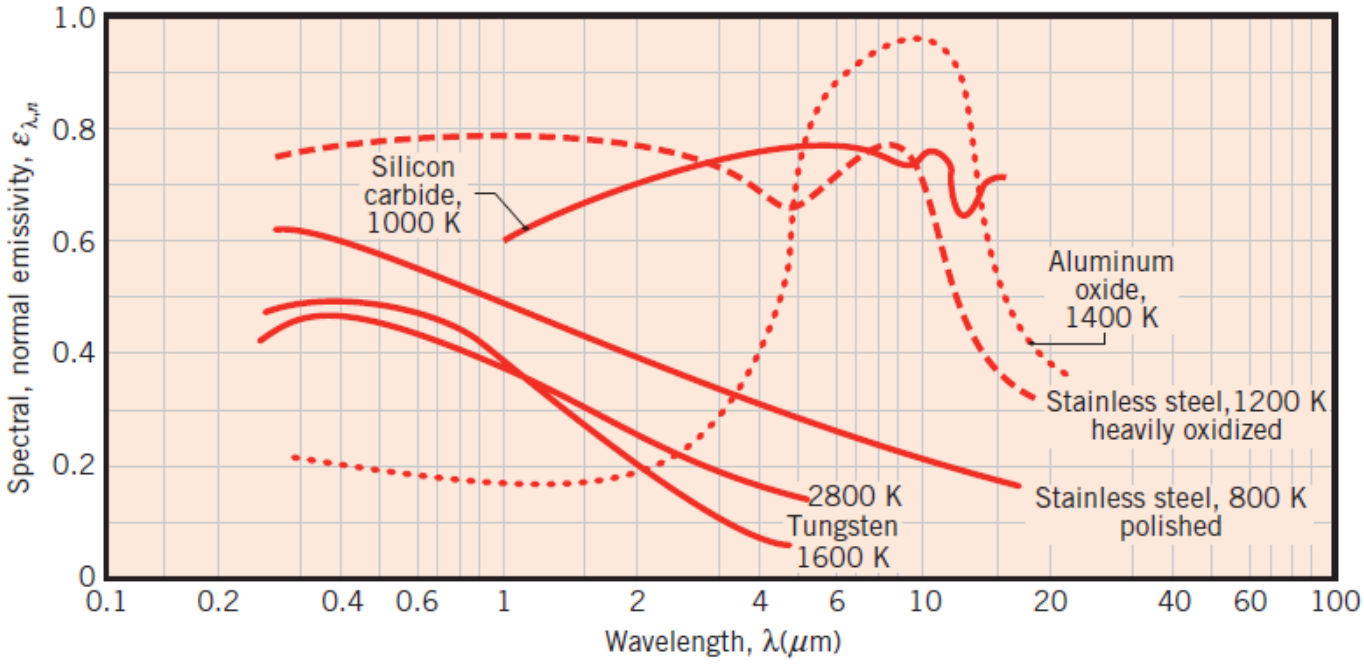


FIGURE 12.17 Spectral dependence of the spectral, normal emissivity $\varepsilon_{\lambda, n}$ of selected materials

Emission from Real Surface

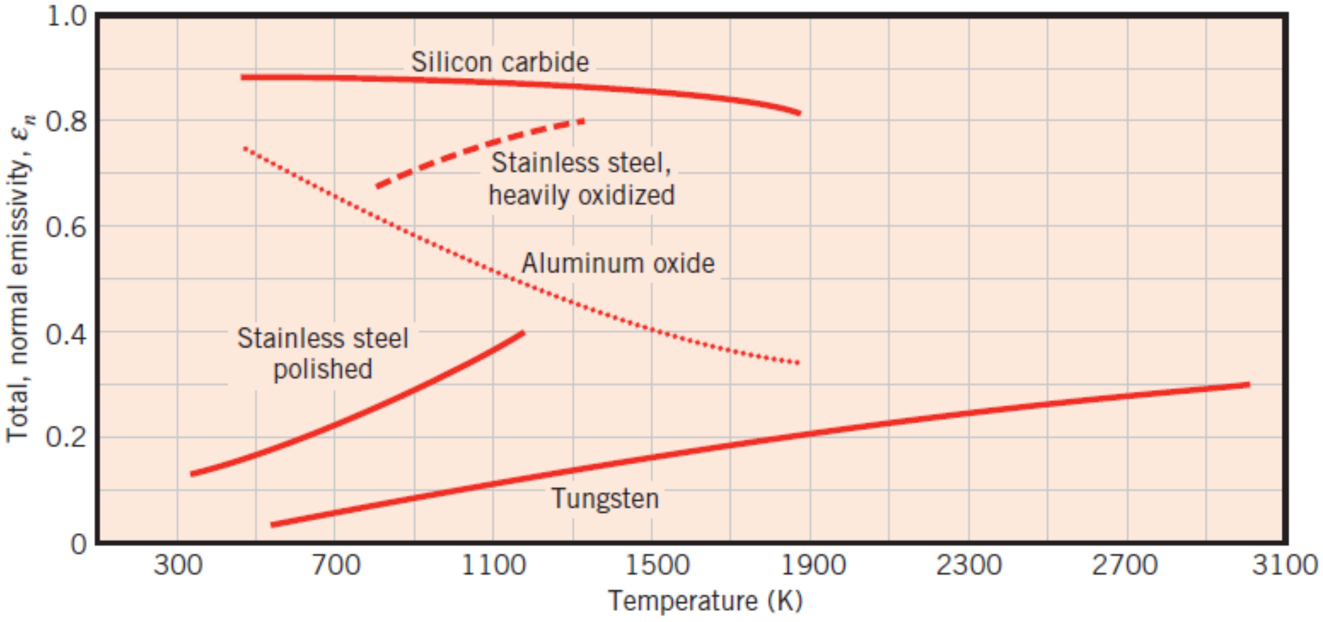
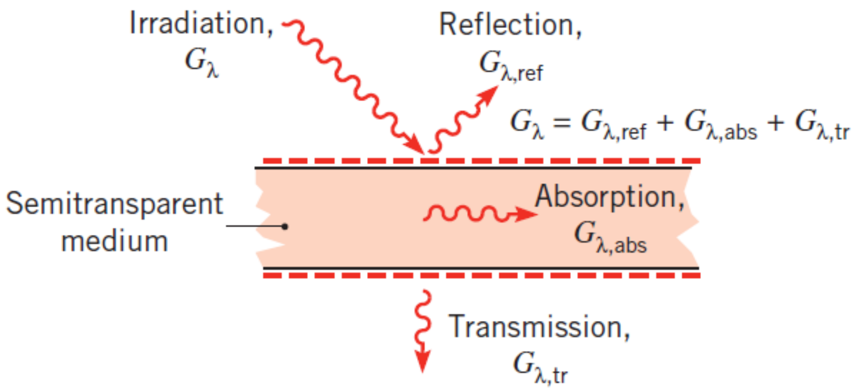


FIGURE 12.18 Temperature dependence of the total, normal emissivity ε_n of selected materials.

Absorption, Reflection & Transmission by Real Surfaces

There may be three responses of a semitransparent medium to irradiation:

- Reflection from the medium $(G_{\lambda,\text{ref}})$.
- Absorption within the medium $(G_{\lambda,\text{abs}})$.
- Transmission through the medium $(G_{\lambda,\text{tr}})$.



Radiation balance: $G_\lambda = G_{\lambda,\text{ref}} + G_{\lambda,\text{abs}} + G_{\lambda,\text{tr}}$
(spectral irradiation)

In contrast to the foregoing volumetric effects, the response of an opaque material $G_{\lambda,\text{tr}} = 0.$ to irradiation is governed by surface phenomena and $G_\lambda = G_{\lambda,\text{ref}} + G_{\lambda,\text{tr}}$

The wavelength of the incident radiation, as well as the nature of the material, determine whether the material is semitransparent or opaque.

Absorptivity

The absorptivity is a property that determines the fraction of the irradiation absorbed by a surface. The *spectral, directional absorptivity*, $\alpha_{\lambda,\theta}$ (λ, θ, ϕ), of a surface is defined as the fraction of the spectral intensity incident in the direction of θ and ϕ that is absorbed by the surface. Hence

The spectral, directional absorptivity: assuming negligible temperature dependence,

$$\alpha_{\lambda,\theta}(\lambda,\theta,\phi) \equiv \frac{I_{\lambda,i,\text{abs}}(\lambda,\theta,\phi)}{I_{\lambda,i}(\lambda,\theta,\phi)}$$

The spectral, hemispherical absorptivity:

$$\alpha_{\lambda}(\lambda) \equiv \frac{G_{\lambda,\text{abs}}(\lambda)}{G_{\lambda}(\lambda)} = \frac{\int_0^{2\pi}\int_0^{\pi/2} \alpha_{\lambda,\theta}(\lambda,\theta,\phi) I_{\lambda,i}(\lambda,\theta,\phi)\cos\theta\sin\theta d\theta d\phi}{\int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,i}(\lambda,\theta,\phi)\cos\theta\sin\theta d\theta d\phi}$$

The total, hemispherical absorptivity:

$$\alpha \equiv \frac{G_{\text{abs}}}{G} = \frac{\int_o^{\infty} \alpha_{\lambda}(\lambda) G_{\lambda}(\lambda) d\lambda}{\int_0^{\infty} G_{\lambda}(\lambda) d\lambda}$$

Reflectivity

The spectral, directional reflectivity: assuming negligible temperature dependence:

$$\rho_{\lambda,\theta}(\lambda,\theta,\phi) \equiv \frac{I_{\lambda,i,\mathrm{ref}}(\lambda,\theta,\phi)}{I_{\lambda,i}(\lambda,\theta,\phi)}$$

The spectral, hemispherical reflectivity:

$$\rho_{\lambda} \equiv \frac{G_{\lambda,\mathrm{ref}}(\lambda)}{G_{\lambda}(\lambda)} = \frac{\int_0^{2\pi}\int_0^{\pi/2} \rho_{\lambda,\theta}(\lambda,\theta,\phi) I_{\lambda,i}(\lambda,\theta,\phi)\cos\theta\sin\theta d\theta d\phi}{I_{\lambda,i}(\lambda,\theta,\phi)}$$

The total, hemispherical reflectivity:

$$\rho \equiv \frac{G_{\mathrm{ref}}}{G} = \frac{\int_0^{\infty} \rho_{\lambda}(\lambda) G_{\lambda}(\lambda) d\lambda}{\int_0^{\infty} G_{\lambda}(\lambda) d\lambda}$$

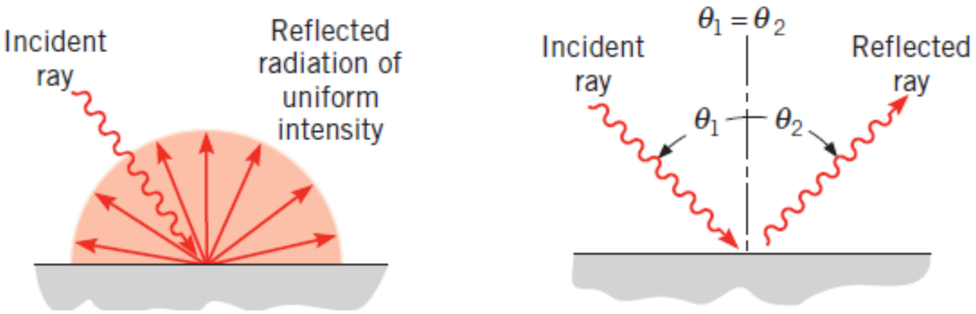


FIGURE 12.21 Diffuse and specular reflection.

Absorptivity & Reflectivity

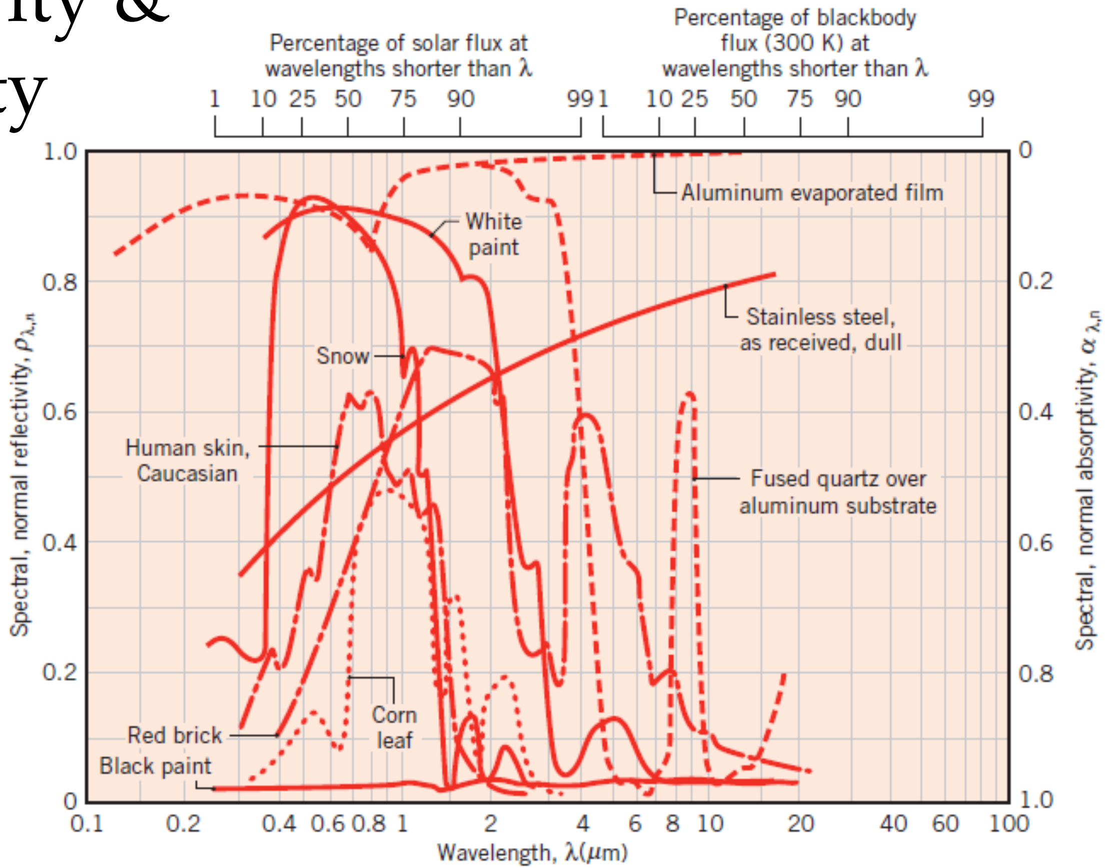


FIGURE 12.22 Spectral dependence of the spectral, normal absorptivity $\alpha_{\lambda,n}$ and reflectivity $\rho_{\lambda,n}$ of selected opaque materials.

Transmission

For a semitransparent medium,

$$\rho_\lambda + \alpha_\lambda + \tau_\lambda = 1$$
$$\rho + \alpha + \tau + 1$$

The spectral, hemispherical transmissivity: assuming negligible temperature dependence,

$$\tau_\lambda \equiv \frac{G_{\lambda,\mathrm{tr}}(\lambda)}{G_\lambda(\lambda)}$$

The total, hemispherical transmissivity:

$$\tau \equiv \frac{G_{\mathrm{tr}}}{G} = \frac{\int_0^\infty G_{\lambda,\mathrm{tr}}(\lambda)\,d\lambda}{\int_0^\infty G_\lambda(\lambda)\,d\lambda}$$

Transmission

Note shift from semitransparent to opaque conditions at large and small wavelengths.

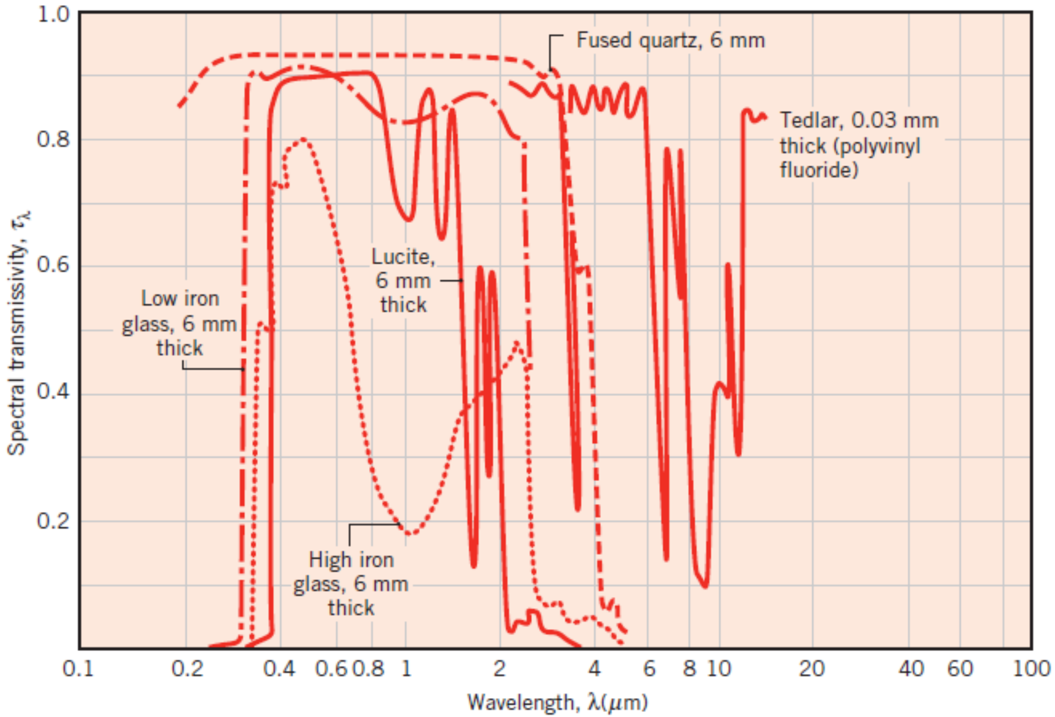


FIGURE 12.23 Spectral dependence of the spectral transmissivities τ_λ of selected semitransparent materials.

Kirchhoff's Law

Kirchhoff's law equates the total, hemispherical emissivity of a surface to its total, hemispherical absorptivity: $\varepsilon = \alpha$
if isothermal conditions exist and and no net radiation heat transfer occurs at any of surfaces

However, conditions associated with its derivation are highly restrictive:
Irradiation of the surface corresponds to emission from a blackbody at the same temperature as the surface.

But, Kirchhoff's law may be applied to the spectral, directional properties without restriction: $\varepsilon_{\lambda,\theta} = \alpha_{\lambda,\theta}$

"Regardless of its orientation, the irradiation experienced by any (small) body in the cavity is diffuse and equal to emission from a blackbody at T_s."

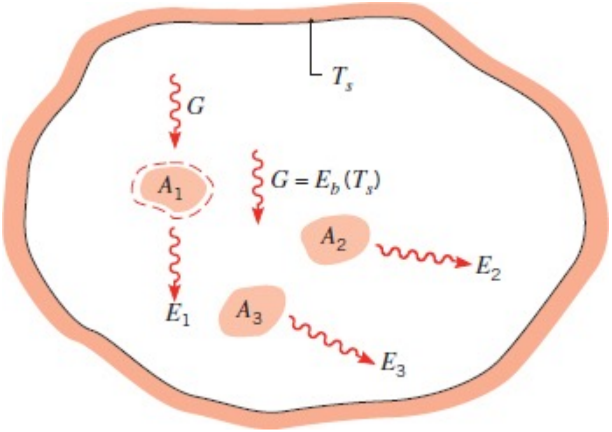


FIGURE 12.24 Radiative exchange in an isothermal enclosure.

The Gray Surface

Defined as "a surface that has an emissivity that is independent of direction (θ, *and* ϕ) and wavelength (λ)"

$$\varepsilon_\lambda = \frac{\int_0^{2\pi}\int_0^{\pi/2} \varepsilon_{\lambda,\theta} \cos\theta \sin\theta d\theta d\phi}{\int_0^{2\pi}\int_0^{\pi/2} \cos\theta \sin\theta d\theta d\phi} \qquad \alpha_\lambda = \frac{\int_0^{2\pi}\int_0^{\pi/2} \alpha_{\lambda,\theta} I_{\lambda,i} \cos\theta \sin\theta d\theta d\phi}{\int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,i} \cos\theta \sin\theta d\theta d\phi}$$

$\varepsilon_\lambda = \alpha_\lambda$ is applicable if *either* of the following conditions is satisfied:

1. The *irradiation* is *diffuse* ($I_{\lambda, i}$ is independent of θ and φ).
2. The *surface* is *diffuse* ($\varepsilon_{\lambda, \theta}$ and $a_{\lambda, \theta}$ are independent of θ and φ).

$$\varepsilon = \frac{\int_0^\infty \varepsilon_\lambda E_{\lambda,b}(\lambda) d\lambda}{E_b(T)} \qquad \alpha = \frac{\int_0^\infty \alpha_\lambda G_\lambda(\lambda) d\lambda}{G}$$

$\varepsilon = \alpha$ applies if either of the following conditions is satisfied:

1. The *irradiation* corresponds to emission from a blackbody at the surface temperature T, in which case $G_\lambda(\lambda) = E_{\lambda,b}(\lambda, T)$ and $G = E_b(T)$.
2. The surface is *gray* (α_λ and ε_λ are independent of λ).

Environmental Radiation - Solar

The sun is a nearly spherical source of radiation whose outer diameter is 1.39×10^9 m and whose emissive power approximates that of a blackbody at 5800 K.
The distance from the center of the sun to the center of the earth varies with time of year from a minimum of 1.471×10^{11} m to a maximum of 1.521×10^{11} m, with an annual average of 1.496×10^{11} m.

Due to the large sun-to-earth distance, the sun's rays are nearly parallel at the outer edge of the earth's atmosphere, and the corresponding radiation flux is $q''_S = f \times S_c$

$S_c \rightarrow$ the solar constant or heat flux $\left(1368\ \mathrm{W/m^2}\right)$ when the earth is at its mean distance from the sun.

$f \rightarrow$ correction factor accounting for eccentricity of the earth's orbit $\left(0.97 < f < 1.03\right)$

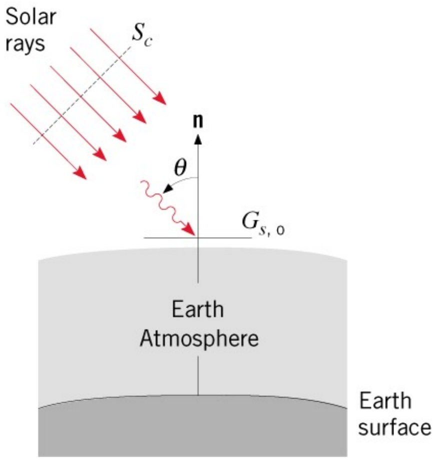
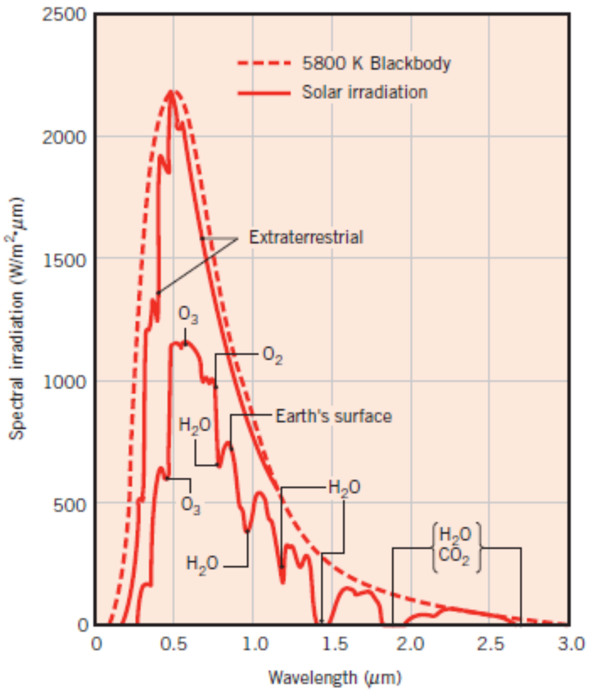
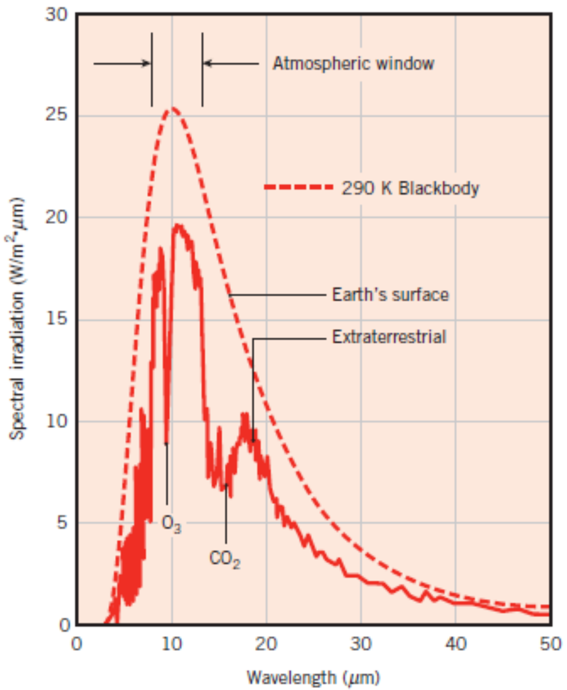


FIGURE 12.27 Directional nature of solar radiation outside the earth's atmosphere.

Extraterrestrial irradiation of a surface whose normal is at a zenith angle θ relative to the sun's rays is $G_{S,o} = f \times S_c \times \cos\theta$

Environmental Radiation – Solar & Atmosphere





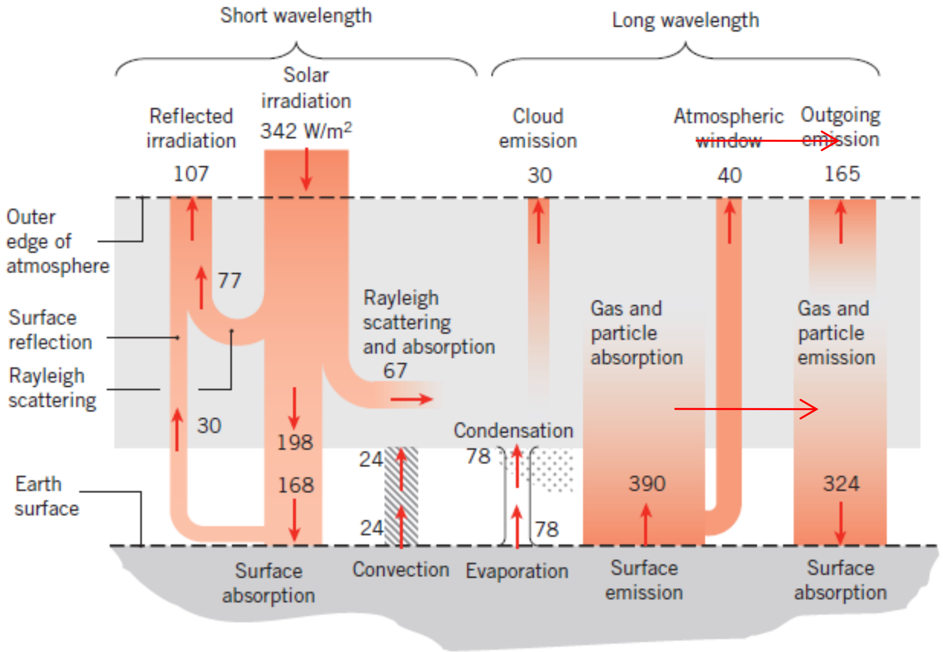
Solar radiation (short wavelengths)

Extraterrestrial solar irradiation is modified spectrally by absorption and scattering by atmospheric components.
Irradiation at Earth's surface is less than at the top of the atmosphere.

Earth emission (long wavelengths)

Emission from Earth's surface is similar to that of a blackbody at 290 K.
Emission is modified spectrally by absorption and scattering by atmospheric components.
Emission at the top of the atmosphere is less than at Earth's surface.

Environmental Radiation - Solar & Atmosphere



An equilibrium energy balance. Heat fluxes are both surface- and time-averaged.
If the chemical constituents of the atmosphere change, atmospheric absorption and scattering will change, potentially resulting in net heating or cooling of the atmosphere.
If the chemical constituents of the atmosphere change, radiation fluxes will change, and surface convection and condensation heat fluxes may be affected. Weather patterns may change.

Environmental Radiation - Atmosphere

Interaction of solar radiation with earth's atmosphere:

- Absorption by aerosols over the entire spectrum.
- Absorption by gases (CO_2, H_2O (υ), O_3) in discrete wavelength bands.
- Scattering by gas molecules and aerosols.

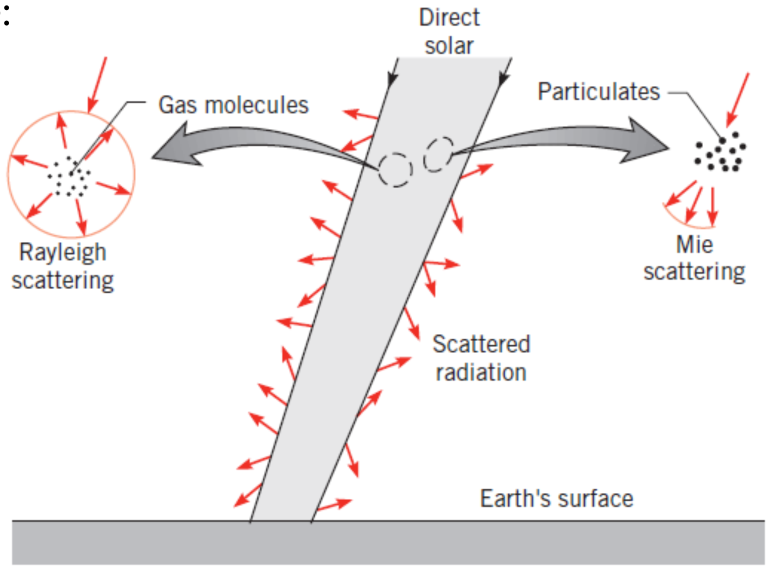


FIGURE 12.29 Scattering of solar radiation in the earth's atmosphere

Environmental Radiation – Terrestrial Solar Irradiation

Emission by Earth's Surface: $E = \varepsilon\sigma T^4$

Emission is typically from surfaces with temperatures in the range of $250 < T < 320$ K and hence concentrated in the spectral region $4 < \lambda < 40\mu m$ with peak emission at $\lambda \approx 10\mu m$.

Although far from exhibiting the spectral characteristics of blackbody emission, earth irradiation due to atmospheric emission is often approximated by a blackbody emissive power of the form

$$G_{atm} = \sigma T_{sky}^4$$

$230 \text{ K} < T_{sky} < 285 \text{ K}$

↳ Cold, clear sky ↳ Warm, overcast sky

$T_{sky} \rightarrow$ the effective sky temperature

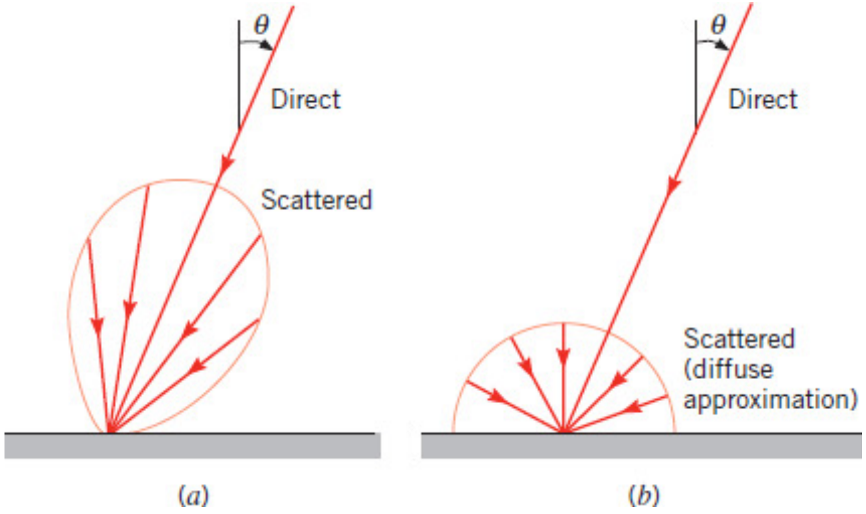


FIGURE 12.30 Directional distribution of solar radiation at the earth's surface. (a) Actual distribution. (b) Diffuse approximation.

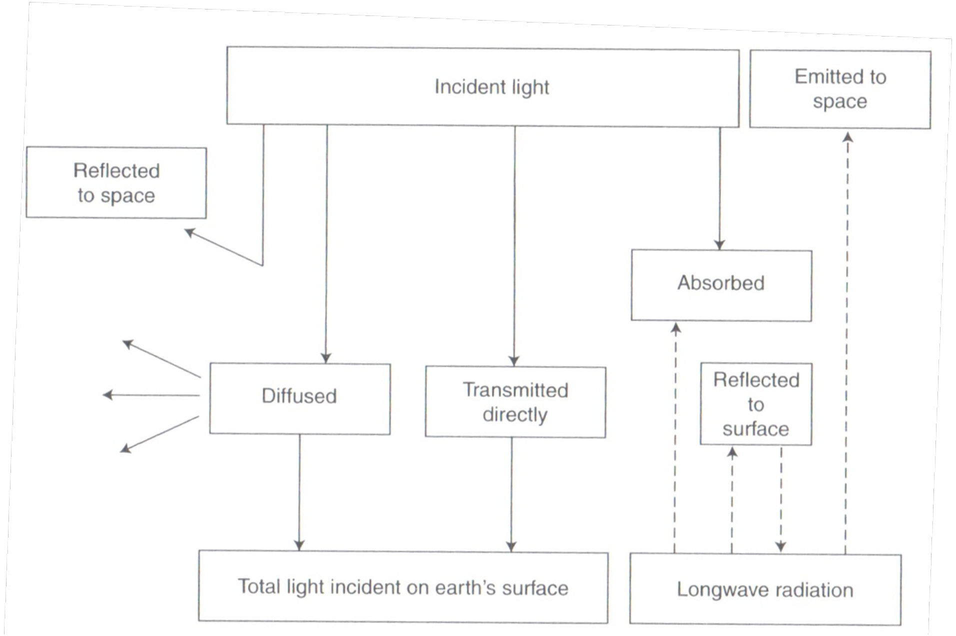
Example 12.12

Climate Model

Climate Model

Carbon Cycle and Solar Radiation

- Carbon in atmosphere is 0.003% of total carbon in earth including earth's crust (99%~$2x10^7$Gt).
- Carbon (7~8Gt per year) added into atmosphere due to fossil fuel.



Climate Model

Gas molecules have different heat absorption capability at specific wavelengths.

Gas	Mass Percent	Contribution to Warming (W/m^2)	Wavelength Range for Peak Contribution* (10^{-6} m)
H_2O	0.23%	~100	>20
CO_2	0.051%	~50	12–14
Other (CH_4, N_2O, O_3, CFC11, CFC12, and so on)	<0.001% for each	<2 for each	For N_2O: 16 For CFCs: 11

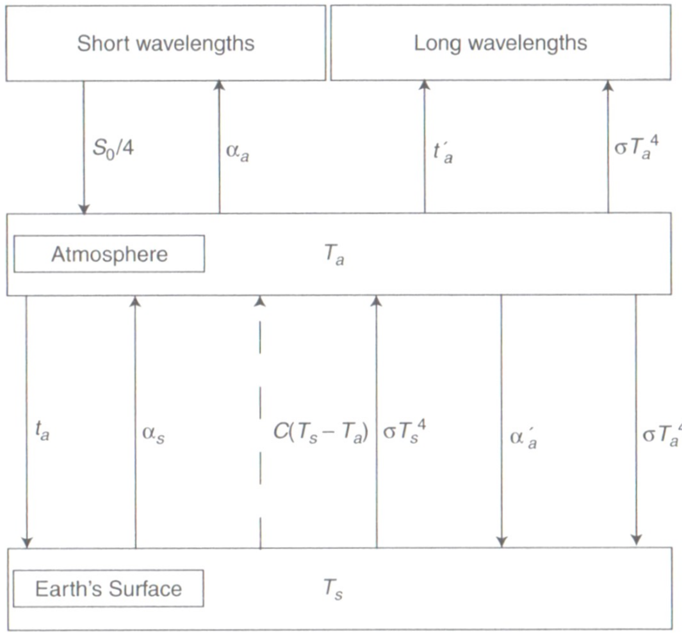


Diagram of simple earth-atmosphere climate model.

Climate Model

A Model of the Earth-Atmosphere System (incl. atmosphere)

Energy Equilibrium for Surface

$$t_a\left(1-\alpha_s\right)S_0/4 - C\left(T_s - T_a\right) - \sigma T_s^4\left(1-\alpha_a^{'}\right) + \sigma T_a^4 = 0$$

1st term: Incoming energy flux from the sun
2nd term: Conductive heat transfer
3rd term: Radiative heat transfer outward
4th term: Radiative heat transfer inward

Energy Equilibrium for Atmosphere

$$\left[1-\alpha_a - t_a\left(1-\alpha_s\right)\right]S_0/4 - C\left(T_s - T_a\right) - \sigma T_s^4\left(1-t_a^{'} - \alpha_a^{'}\right) + 2\sigma T_a^4 = 0$$

1st term: Incoming energy flux from solar gain
2nd term: Conductive heat transfer
3rd term: Radiative heat transfer inward from earth’s surface
4th term: Radiative heat transfer outward to surface and space

Climate Model

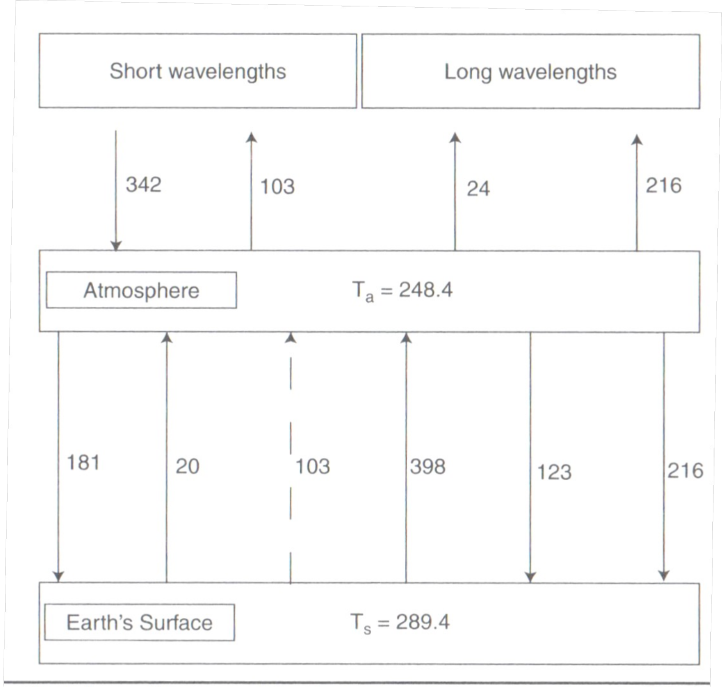
A Model of the Earth-Atmosphere System (incl. atmosphere)

The results of solving the system of equations using the initial parameter value.

For 1980~2000, earth temperature was 288K.

Energy balance excluding atmosphere predicted 255K.

Inclusion of atmosphere can provide much closer prediction of surface temperature to the actual value.



Single-layer climate model with representative energy flows in W/m².

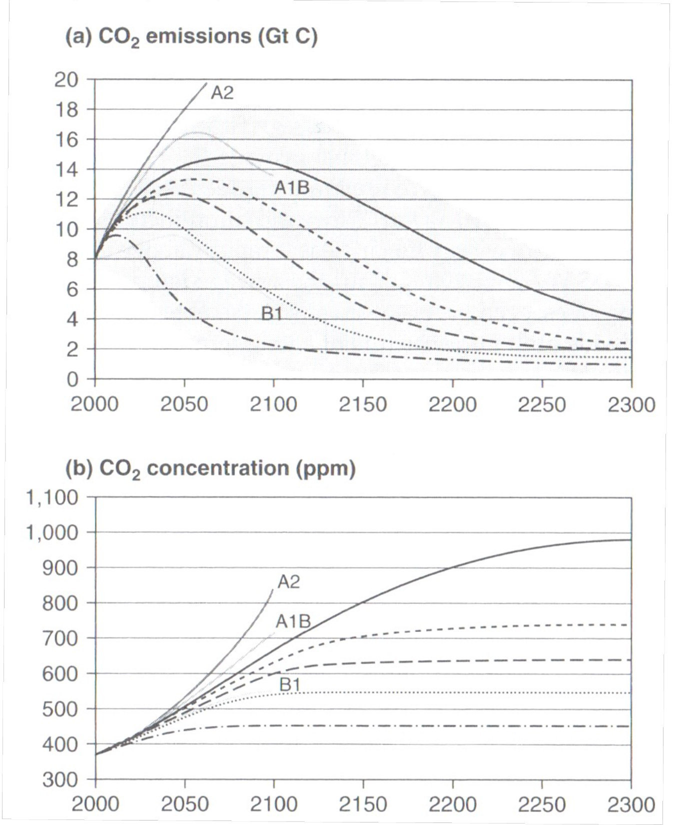
Climate in the Future

Scenarios for Future Rates of CO2 Emission, Stabilization Values and Average Global Temperature

<u>Possible pathways for CO2 emissions</u>
A1B=rapid economic growth & peaking population with balance of energy sources.

A2=continued population growth with widely divergent economic growth bwtn the regions.

B1=similar to A1B but transition to less materially intensive energy with cleaner energy sources

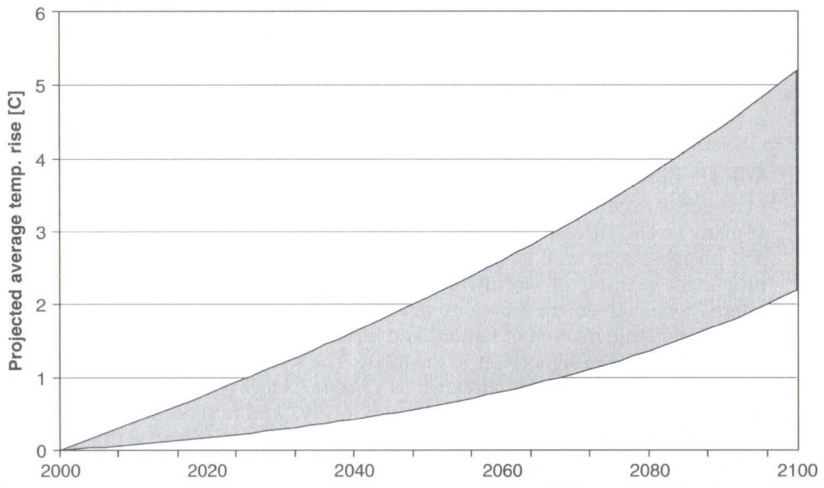


Climate in the Future

Scenarios for Future Rates of CO2 Emission, Stabilization Values and Average Global Temperature

The trends are agreed among GCMs and also for the rise of averaged temperature by 2°C at least.

However, rise of averaged temperature by 5°C is not certain.



Long Term Effects

- Sea water: 1 degree rise of sea water temperature will raise the sea level by 2.5cm due to its density change.
- Ice in Antarctica and Greenland: $3x10^{16}$ tons and ~2% of ocean
- Low-lying areas are in danger
- By IPCC, reported importance of stabilizing atmospheric concentration of CO2 at 450ppm and temperature rise at around 2°C. to avoid severe long term effect.

Chapter 13 Radiation Exchange between Surfaces

Scope

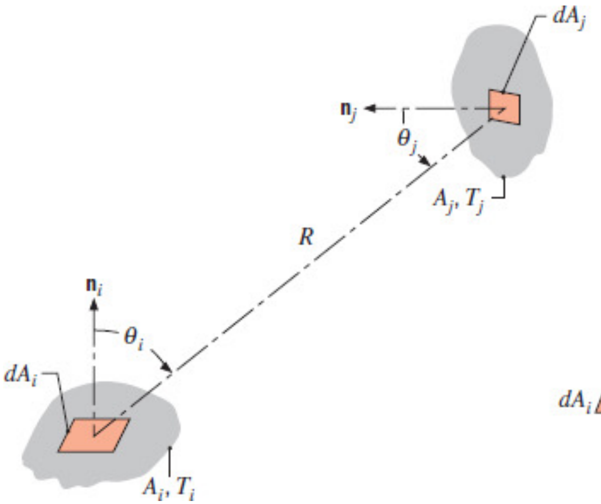
- Radiative exchange between two or more surfaces assuming nonparticipating medium (such as vacuum)
- Learn about (1) View factor and (2) radiative exchange within Enclosure
- Radiative exchange between surfaces by considering the effects of a participating medium

View Factor (Shape Factor)

The view factor F_{ij} is defined as the fraction of the radiation leaving surface *i* that is intercepted by surface *j*.

$$F_{ij} = \frac{q_{i\to j}}{A_i J_i} \qquad A_i F_{ij} = A_j F_{ji} \qquad \sum_{j=1}^{N} F_{ij} = 1$$

where $J = E + G_{ref} = E + \rho G$



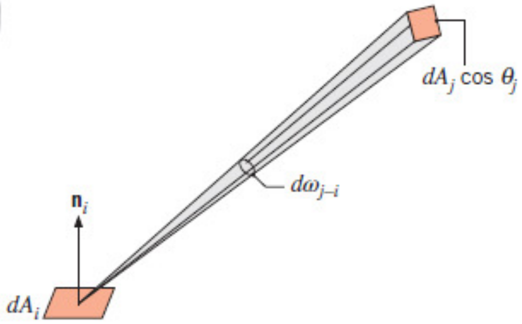


Figure 13.1 View factor associated with radiation exchange between elemental surfaces of area dA_i and dA_j.

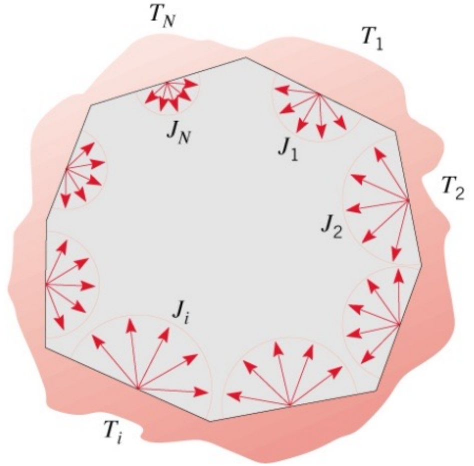


FIGURE 13.2 Radiation exchange in an enclosure.

View Factor (Shape Factor)

The view factor integral provides a general expression for F_{ij}.

$$dq_\lambda \equiv \frac{dq}{d\lambda} = I_{\lambda,e}(\lambda,\theta,\phi)\, dA_1 \cos\theta\, d\omega$$

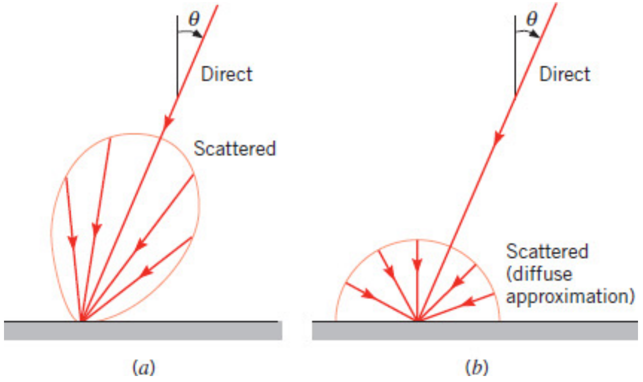


FIGURE 12.30

Consider exchange between diffusely-emitting and reflecting differential areas dA_i and dA_j:

$$dq_{i\to j} = I_{e+r,i} \cos\theta_i dA_i d\omega_{j-i} = J_i \frac{cos\theta_i cos\theta_j}{\pi R^2} dA_j dA_i$$

$$d\omega \approx \frac{dA_n}{r^2} = \frac{(cos\theta_j dA_j)}{R^2}$$

$$J = \pi I_{e+r}$$

$$F_{ij} = \frac{1}{A_i}\int_{A_i}\int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi R^2} dA_j dA_i \quad \Leftarrow \quad F_{ij} = \frac{q_{i\to j}}{A_i J_i}$$

$$F_{ji} = \frac{1}{A_j}\int_{A_i}\int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi R^2} dA_j dA_i$$

Please refer to Table 13.1 View factors for two-dimensional geometries

TABLE 13.1 View factors for two-dimensional geometries.

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<tr><th>Geometry</th><th>Relation</th></tr>
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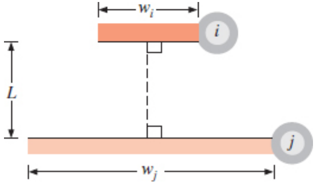
Parallel Plates with Midlines Connected by Perpendicular



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$$F_{ij} = \frac{\left[(W_i + W_j)^2 + 4\right]^{1/2} - \left[(W_j - W_i)^2 + 4\right]^{1/2}}{2W_i}$$

$$W_i = w_i/L, W_j = w_j/L$$

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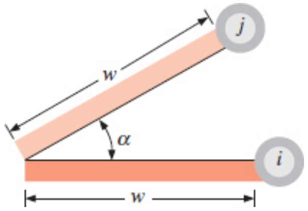
Inclined Parallel Plates of Equal Width and a Common Edge



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$$F_{ij} = 1 - \sin\left(\frac{\alpha}{2}\right)$$

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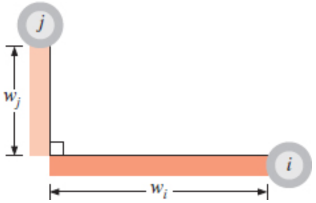
Perpendicular Plates with a Common Edge



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$$F_{ij} = \frac{1 + (w_j/w_i) - \left[1 + (w_j/w_i)^2\right]^{1/2}}{2}$$

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Blackbody Radiation Exchange

$$q_{i\to j} = (A_i J_i) F_{ij}$$

For a blackbody, $J_i = E_{bi}$.

$$q_{i\to j} = A_i F_{ij} E_{bi}$$

$$q_{j\to i} = A_j F_{ji} E_{bj}$$

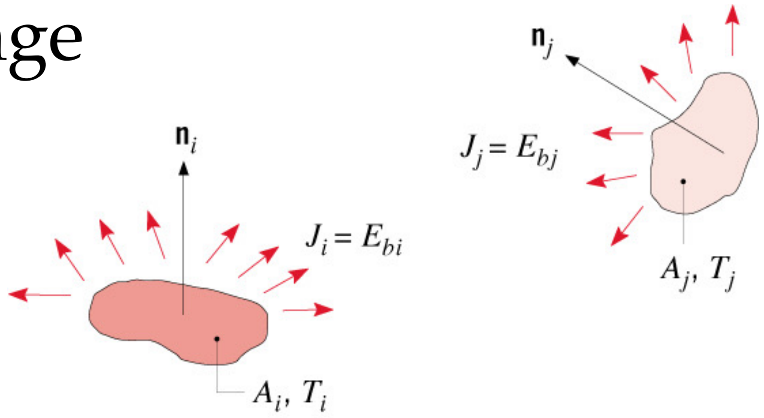


FIGURE 13.8 Radiation transfer between two surfaces that may be approximated as blackbodies.

net rate at which radiation leaves surface *i* due to its interaction with *j*

or net rate at which surface *j* gains radiation due to its interaction with *i*

$$q_{ij} = q_{i\to j} - q_{j\to i}$$

$$q_{ij} = A_i F_{ij} E_{bi} - A_j F_{ji} E_{bj} \Leftarrow A_i F_{ij} = A_j F_{ji}$$

$$q_{ij} = A_i F_{ij} \sigma \left(T_i^4 - T_j^4 \right)$$

$$q_i = \sum_{j=1}^{N} A_i F_{ij} \sigma \left(T_i^4 - T_j^4 \right)$$

Net radiative heat flux: $q_i'' = q_i / A_i = q_{rad}''$

Radiation Exchange between Surfaces in an Enclosure

Alternative expressions for net radiative transfer from surface *i*:

(1) $q_i = A_i\left(J_i - G_i\right) \rightarrow$ Fig. (b) and $J_i = E_i + \rho_i G_i$

(2) $q_i = A_i\left(E_i - \alpha_i G_i\right) \rightarrow$ Fig. (c)

since $J_i = \varepsilon_i E_{bi} + (1-\varepsilon_i)G_i$ ⇨ $q_i = A_i\left(J_i - \dfrac{J_i - \varepsilon_i E_{bi}}{1-\varepsilon_i}\right)$

(3) $q_i = \dfrac{E_{bi} - J_i}{(1-\varepsilon_i)/\varepsilon_i A_i}$

Suggests a surface radiative resistance of the form: $\left(1-\varepsilon_i\right)/\varepsilon_i A_i$

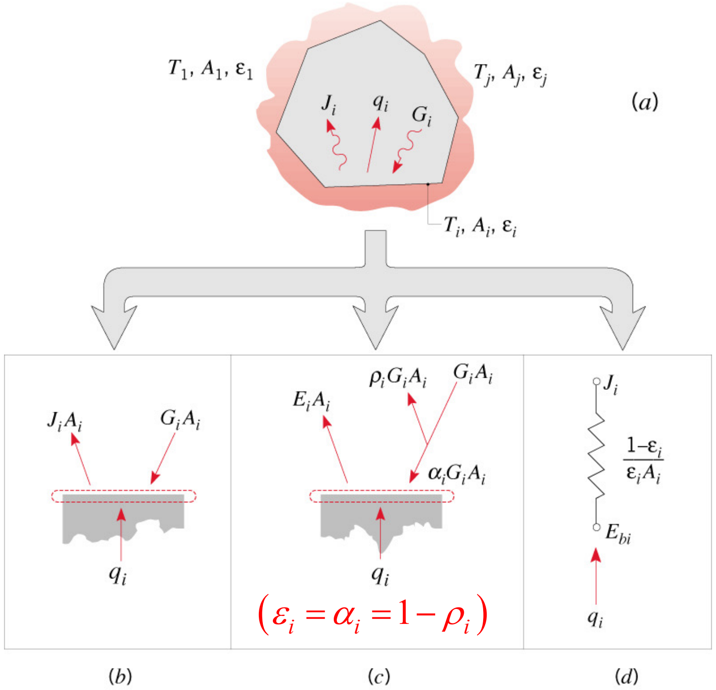


FIGURE 13.9 Radiation exchange in an enclosure of diffuse, gray surfaces with a nonparticipating medium. (a) Schematic of the enclosure. (b) Radiative balance according to Equation 13.15. (c) Radiative balance according to Equation 13.17. (d) Resistance representing net radiation transfer from a surface, Equation 13.19

Radiation Exchange between Surfaces in an Enclosure

$$q_i = \sum_{j=1}^{N} A_i F_{ij}\left(J_i - J_j\right) = \sum_{j=1}^{N} \frac{J_i - J_j}{\left(A_i F_{ij}\right)^{-1}} \qquad (4)$$

Suggests a space or geometrical resistance of the form: $\left(A_i F_{ij}\right)^{-1}$

Equating Eqs. (3) and (4) corresponds to a radiation balance on surface i:

$$\frac{E_{bi} - J_i}{\left(1-\varepsilon_i\right)/\varepsilon_i A_i} = \sum_{j=1}^{N} \frac{J_i - J_j}{\left(A_i F_{ij}\right)^{-1}} \qquad (5)$$

which may be represented by a radiation network of the form

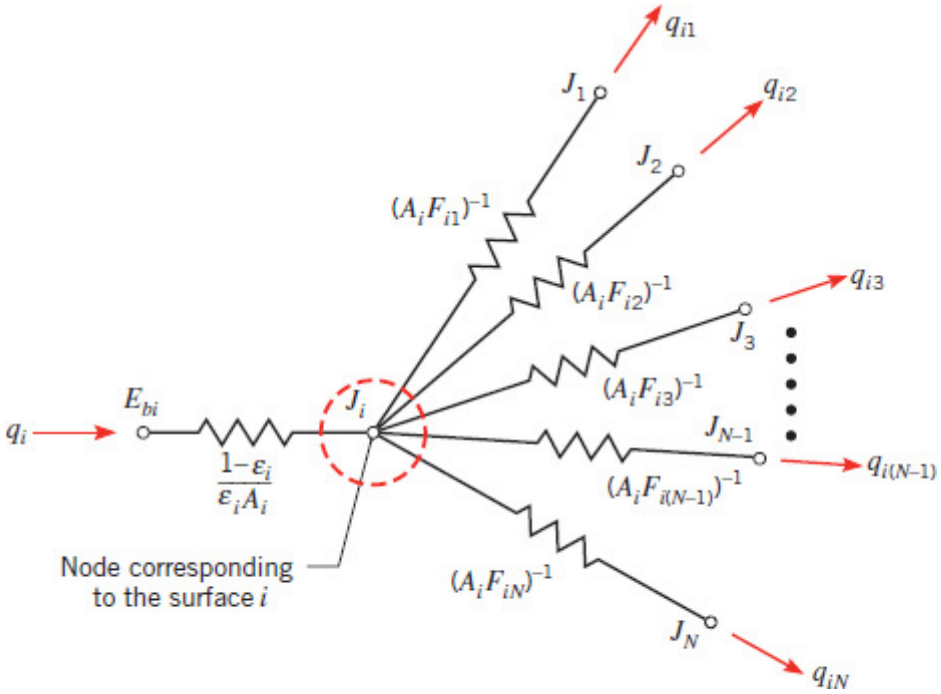


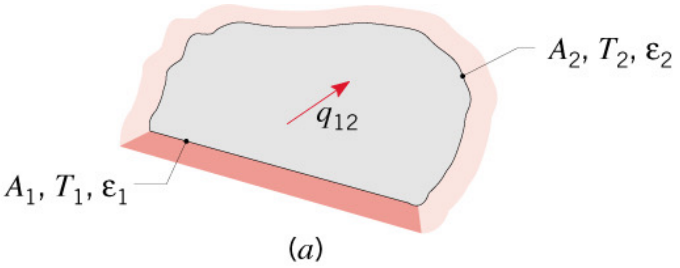
FIGURE 13.10 Network representation of radiative exchange between surface i and the remaining surfaces of an enclosure

Example 13.4

Two-Surface Enclosure

Simplest enclosure for which radiation exchange is exclusively <u>between two surfaces</u> and a single expression for the rate of radiation transfer may be inferred from a network representation of the exchange.

$$q_1 = -q_2 = q_{12} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1 F_{12}} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$



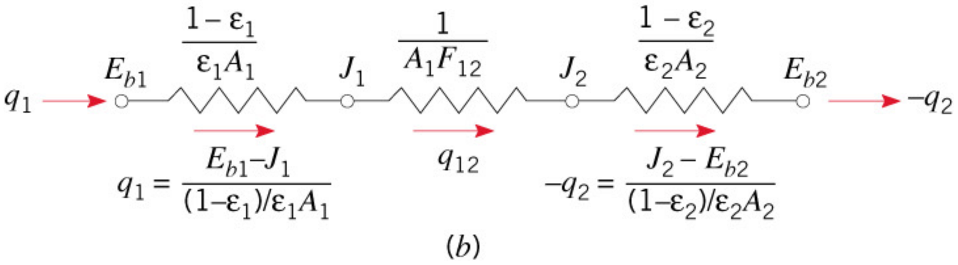
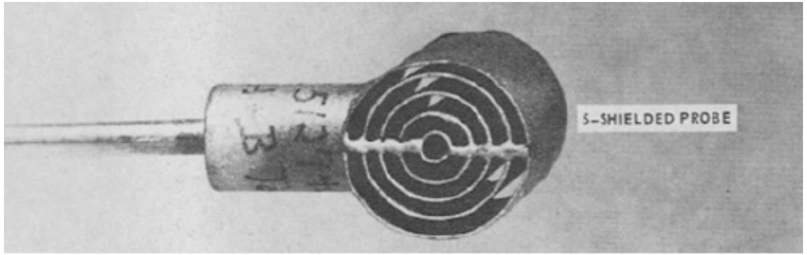


FIGURE 13.11 The two-surface enclosure. (a) Schematic. (b) Network representation.

Two-Surface Enclosure – Radiation Shields

An important application of having multiple two-surface enclosures arranged in series is the use of radiation shields, typically constructed from low emissivity (high reflectivity) materials, to reduce the net radiation transfer between two surfaces.

$$q = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{1s}} + \frac{1-\varepsilon_{s1}}{\varepsilon_{s1} A_s} + \frac{1-\varepsilon_{s2}}{\varepsilon_{s2} A_s} + \frac{1}{A_s F_{s2}} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$



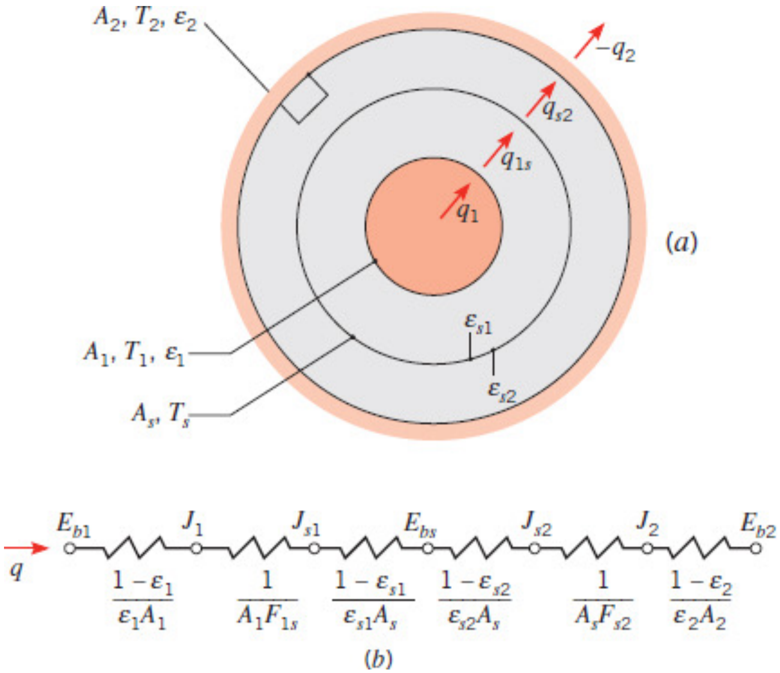


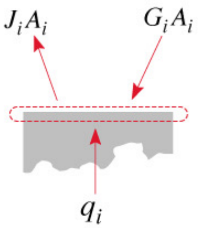
FIGURE 13.12 Radiation exchange between surfaces with a radiation shield in place. (a) Schematic. (b) Network representation.

The Reradiating Surface

Reradiating surface is characterized by zero net radiation transfer, $q_i = A_i(J_i - G_i) = 0$.

It is closely approached by real surfaces that are well insulated on one side and for which convection effects may be neglected on the opposite (radiating) side.

With $q_i = 0$, it follows that $G_i = J_i = E_{bi}$.



$$\frac{J_1 - J_R}{(1/A_1F_{1R})} = \frac{J_R - J_2}{(1/A_2F_{2R})}$$

where $J_R = \sigma T_R^4$

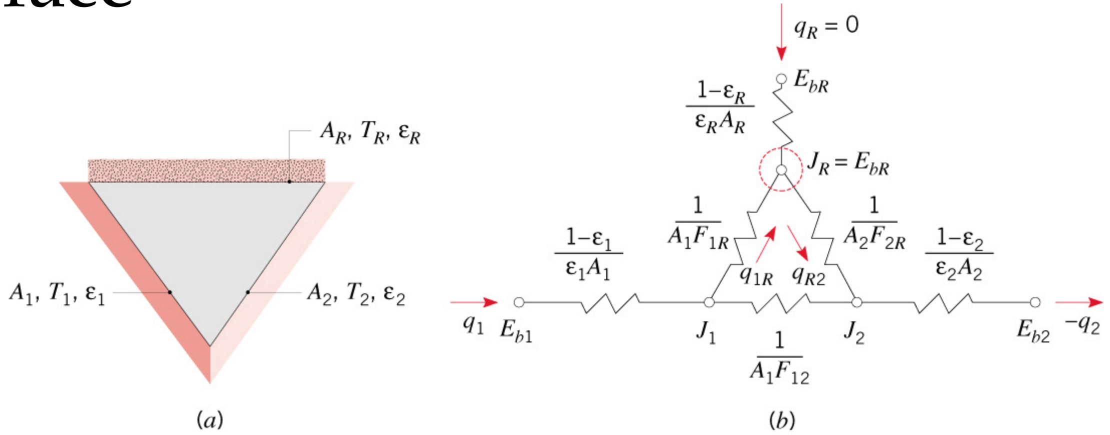


FIGURE 13.13 A three-surface enclosure with one surface reradiating. (a) Schematic. (b) Network representation.

$$q_1 = -q_2 = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1F_{12} + \left[(1/A_1F_{1R}) + (1/A_2F_{2R})\right]^{-1}} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

Multimode Heat Transfer

In an enclosure with conduction and convection heat transfer to or from one or more surfaces, the foregoing treatments of radiation exchange may be combined with surface energy balances to determine thermal conditions.

Consider a general surface condition for which there is external heat addition (e.g., electrically), as well as conduction, convection and radiation.

$$q_{i,\text{ext}} = q_{i,\text{rad}} + q_{i,\text{conv}} + q_{i,\text{cond}}$$

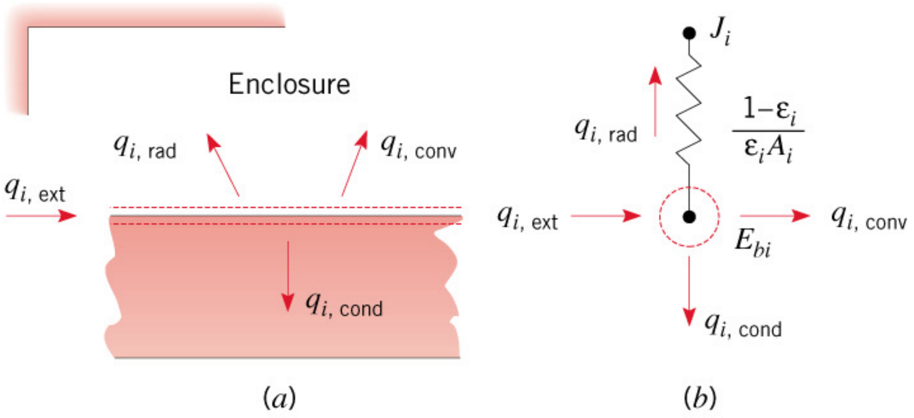


FIGURE 13.14 Multimode heat transfer from a surface in an enclosure. (a) Surface energy balance. (b) Circuit representation.

Implications of the Simplifying Assumptions

We have assumed idealized surfaces:

- Isothermal, Opaque, and gray surfaces.
- Diffusely emitting and reflecting surfaces.
- Surfaces that all experience uniform irradiation.
- Surfaces that all produce uniform radiosity.
- Enclosures with gases that do not emit, absorb, or scatter.

Seldom are all the assumptions rigorously satisfied.

The analysis technique may, however, be used to obtain first estimates.

The implications of not satisfying all the assumptions rigorously are often less severe in problems involving multimode effects.

Radiation Exchange with Participating Media

The medium separating surfaces of an enclosure may affect radiation at each surface through its ability to absorb, emit and/or scatter (redirect) radiation.

The gaseous radiation is concentrated in specific wavelength intervals (called bands) and is a volumetric phenomenon.

Polar molecules, such as CO_2, H_2O (vapor), NH_3, and hydrocarbon gases emit and absorb over a wide temperature range.

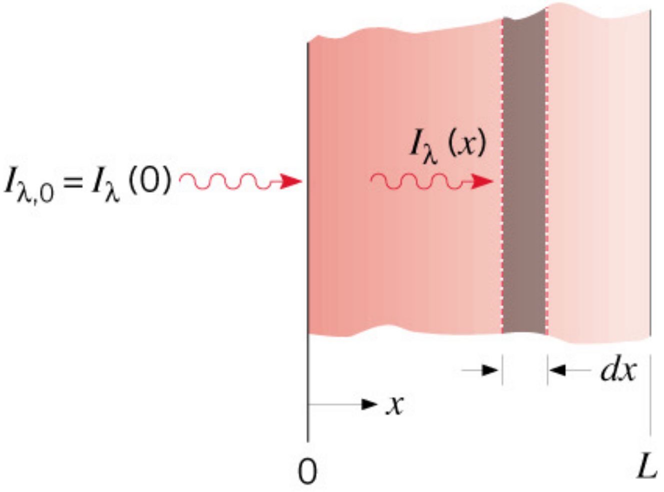


FIGURE 13.15 Absorption in a gas or liquid layer.

Radiation Exchange with Participating Media

Beer's law: A simple relation for predicting the exponential decay of radiation propagating through an absorbing medium.

$$dI_\lambda(x) = -\kappa_\lambda I_\lambda(x)dx$$

$\kappa_\lambda \rightarrow$ spectral absorption coefficient (1/m)

$$\int_{I_{\lambda,0}}^{I_{\lambda,L}} \frac{dI_\lambda(x)}{dI_\lambda} = -\kappa_\lambda \int_0^L dx$$

$$\frac{I_{\lambda,x}}{I_{\lambda,0}} = \exp(-\kappa_\lambda x)$$

Transmissivity and absorptivity of medium of thickness L:

$$\tau_\lambda = \left(I_{\lambda,L} / I_{\lambda,0}\right) = \exp\left(-\kappa_\lambda L\right) \qquad \alpha_\lambda = 1 - \tau_\lambda = 1 - \exp\left(-\kappa_\lambda L\right)$$

Assuming applicability of Kirchhoff's law (gray surface): $\varepsilon_\lambda = \alpha_\lambda$

Gaseous Emission and Absorption

$$E_g = \varepsilon_g \sigma T_g^4$$

where, ε_g (emissivity)=$f(T_g, P_{total}, p_g, L)$

T_g = gas temperature
L = radius of hemisphere
ε_w = emissivity of water vapor
p_w, p_c = the partial pressures of H_2O (water vapor) and CO_2(carbon dioxide)

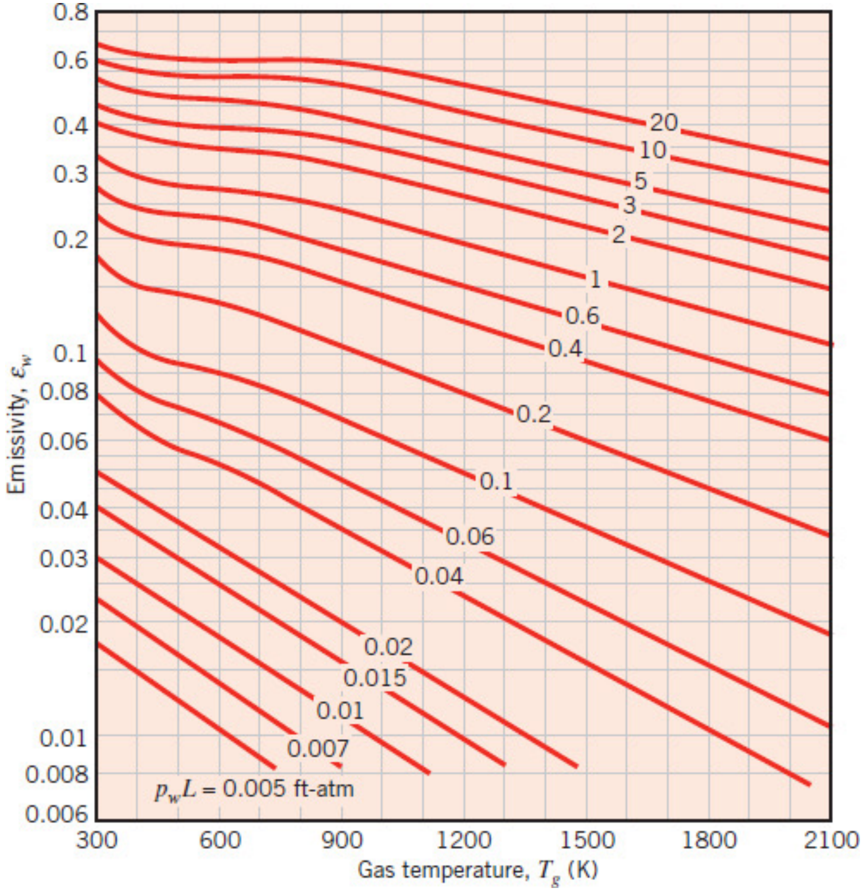
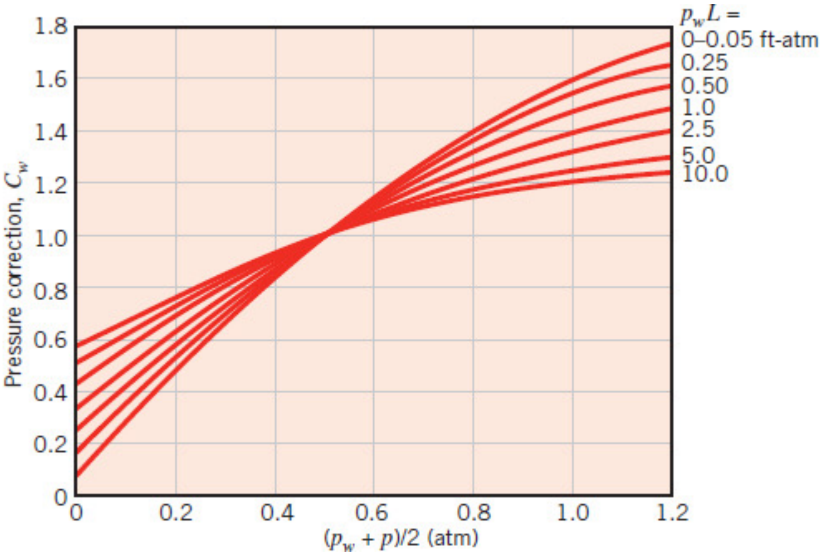


FIGURE 13.16 Emissivity of water vapor in a mixture with nonradiating gases at 1-atm total pressure and of hemispherical shape [13].

Gaseous Emission and Absorption



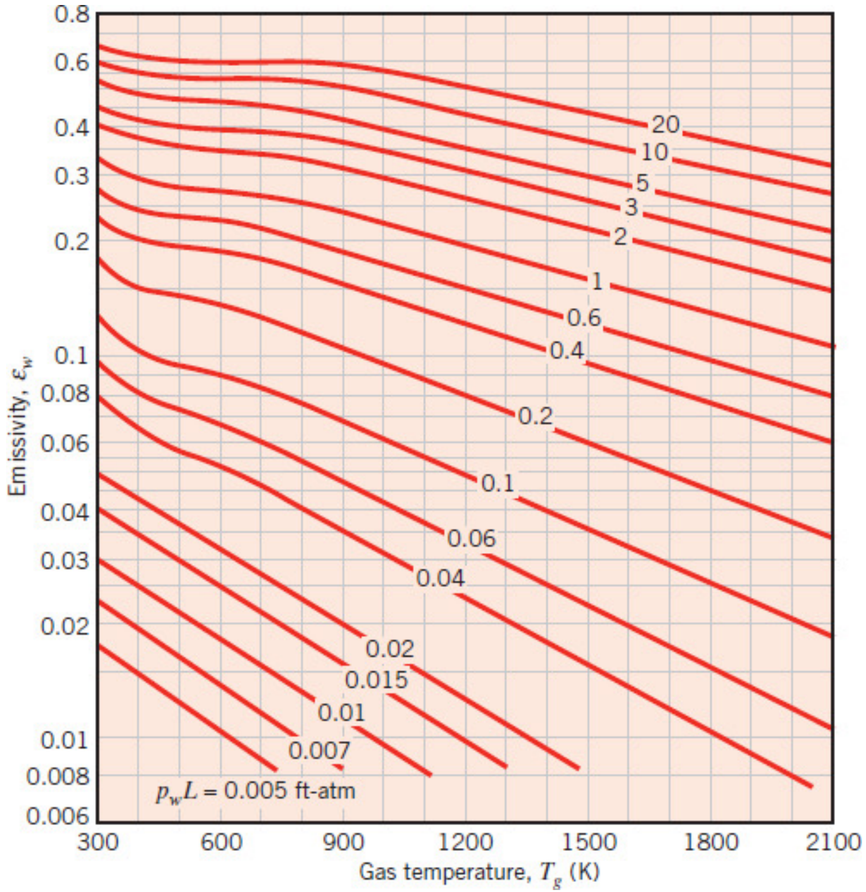
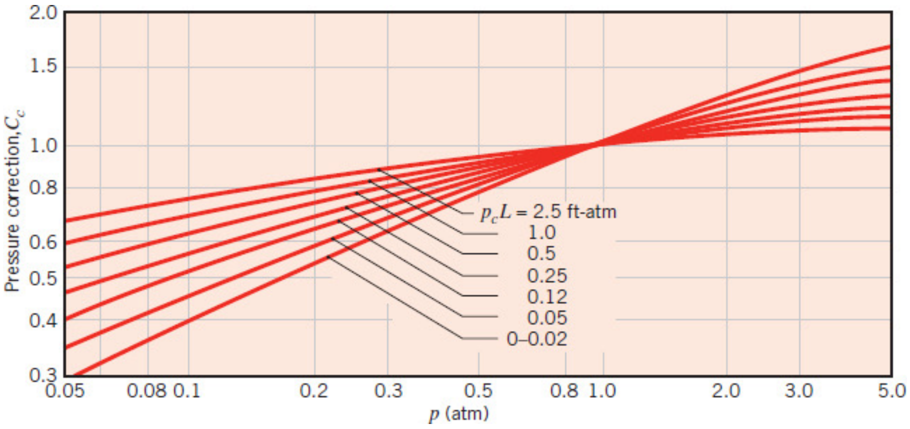


FIGURE 13.17 Correction factor for obtaining water vapor emissivities at pressures other than $\left(\varepsilon_{w,p\neq 1atm} = C_w \varepsilon_{w,p=1atm}\right)$[13]

Gaseous Emission and Absorption



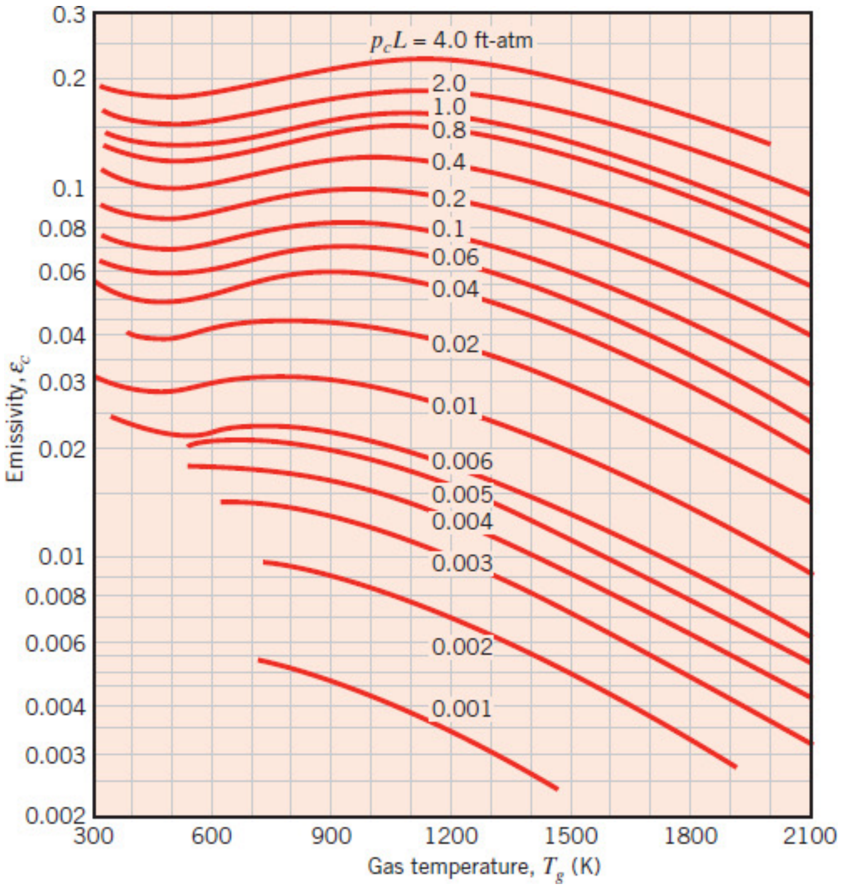


FIGURE 13.19 Correction factor for obtaining carbon dioxide emissivities at pressures other than 1 atm $\left(\varepsilon_{c,p\neq 1atm} = C_c \varepsilon_{c,p=1atm}\right)$[13]

Gaseous Emission and Absorption

$$\varepsilon_g = \varepsilon_w + \varepsilon_c - \Delta\varepsilon$$

$\Delta\varepsilon \rightarrow$ correction factor for mutual absorption of radiation for H_2O (water vapor) and CO_2(carbon dioxide)

For application to other gas geometries, replace L by L_e (radius of a hemispherical gas mass whose emissivity, is equivalent to that for the geometry of interest).

$L_e \rightarrow$ Table 13.4

Rate of heat transfer to a surface of area A_s due to emission from an adjoining gas is

$$q = \varepsilon_g A_s \sigma T_g^4$$

Net rate of radiation exchange between a black surface and an adjoining gas is

$$q_{net} = A_s \sigma \left(\varepsilon_g T_g^4 - \alpha_g T_s^4 \right)$$

$$\alpha_g = \alpha_w + \alpha_c - \Delta\alpha$$

TABLE 13.4 Mean beam lengths L_e for various gas geometries

Geometry	Characteristic Length	L_e
Sphere (radiation to surface)	Diameter (D)	$0.65D$
Infinite circular cylinder (radiation to curved surface)	Diameter (D)	$0.95D$
Semi-infinite circular cylinder (radiation to base)	Diameter (D)	$0.65D$
Circular cylinder of equal height and diameter (radiation to entire surface)	Diameter (D)	$0.60D$
Infinite parallel planes (radiation to planes)	Spacing between planes (L)	$1.80L$
Cube (radiation to any surface)	Side (L)	$0.66L$
Arbitrary shape of volume V (radiation to surface of area A)	Volume to area ratio (V/A)	$3.6V/A$